



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Health and Applied Sciences

DEPARTMENT MATHS AND PHYSICS

**UNIVERSITY EXAMINATIONS FOR DEGREE IN
BACHELOR OF SCIENCE CIVIL ENGINEERING**

SMA 2371: PDE

END OF SEMESTER EXAMINATIONS

SERIES: APRIL 2014

TIME: 2 HOURS

INSTRUCTIONS:

- You should have the following for this examination:
 - Answer booklet
 - Calculator
- Answer question **ONE (Compulsory)** and any other **TWO**.

This paper consists of Three printed pages

QUESTION 1 (Compulsory)

- a) Verify that $u = F(x - ct) + 9(x + ct)$ is a solution of the one-dimensional wave equation $utt = c^2 uxx$. **(5 marks)**
- b) Obtain the solution to the following initial value problem $u_{xx} = 4xy + e^x$ with initial conditions $u(0, y) = y$, $u_x(0, y) = 1$ **(5 marks)**
- c) Find the Integral curves of the equations:
$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$$
 (6 marks)
- d) Classify the following equation and hence determine its characteristics of the differential equation.

$$4uxx + u_yy = 0 \quad (6 \text{ marks})$$

e) Find the fourier transform of

$$f(x) = e^{-ax^2} \quad a > 0$$

(8 marks)

QUESTION 2

a) i) Classify the langranses equation $(x^2 - 1)p + xyq + y^2z = x^3 - 1$ where P and Q have their visual meaning as either Linear, Semi linear or Quasi linear. **(2 marks)**

ii) Use the method of characteristics to find the solution of the equation $(x^2 - 1)p + xyq + y^2z = x^2 - 1$ which is zero on the positive y – axis. **(8 marks)**

b) Using the method of separation of variables solve

$$\frac{du}{d>1} = 2 \frac{dy}{dt} + u \quad \text{where } u(x,0) = 6e^{-3x}$$

(10 marks)

QUESTION 3

a) Test for the integrability of the equation $21t^3 dx - zdz + 2y dz = 0$

(6 marks)

b) Reduce the equation $2uxx + 3uxy + u_yy = 0$ to canonical form.

(14 marks)

QUESTION 4

a) Solve the homogeneous differential equation

$$yz(y + z)dx + xz(x + z)dy + xy(x + y)dz = 0$$

marks)

(10

b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections with the family of planes parallel to $z = c$.

(10 marks)

QUESTION 5

a) Find the surface which intersects with the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passed through the circle $x^2 + y^2 = 1$, $z = 1$

(10 marks)

b) Solve the following equation

$$\frac{d^2z}{dx^2} - 2 \frac{dz}{dx} + \frac{dz}{dy} = 0 \quad \text{by the method of separation of variables.}$$

(10 marks)