TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 5106: TEST OF HYPOTHESIS

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 3 HOURS

DATE: MAY

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of five questions. Attempt any three.
Do not write on the question paper.

Question ONE

a. Define type II error (3marks)

b. If $x \geq 1,\ is\ the\ critical\ region\ for\ testing\ \ H_0: \theta = 2\ against\ H_1: \theta = 1 f(x;\theta) = \theta e^{-\theta x}$ from a single observation. Obtain;

   i. Type I error (4marks)

   ii. Type II error (4marks)

   iii. Power of the test (2marks)

c. Costly road testing of random samples of cars was carried out to determine if mean mileage is greater for model 1 cars than for model 2 cars the sample data were as follows. perform hypothesis test at the 5% level (6marks)

Model 1	$n_1 = 8$	$\bar{x}$=26	$s_1 = 1.4$
Model 1	$n_2 = 8$	$\bar{x}$=23.6	$s_2 = 1.2$

d. Define most powerful test (4marks)

e. Consider $x_1, x_2, \ldots, x_n$ of iid continuous random variables with mean μ and known variance σ^2. We wish to test the hypothesis $H_0: \mu = \mu_0\ against\ H_1: \mu = \mu_1$. obtain the most powerful test (7marks)

Question TWO

a. Let $x_1, x_2,..., x_n$ be a random sample from $f(x;\theta)$ where Ω is some interval. Assume that the family of densities $\{f(x;\theta); \theta\epsilon\Omega\}$ has a monotone likelihood ratio in the statistics $T(x)$. show that if the monotone likelihood ratio is none decreasing in $T(x)$ and k^* is such that $P(T < k^*) = \alpha$ then the test corresponds to the critical region is a UMP test of size α of $H_0: \theta \leq \theta_0$ $against$ $H_0: \theta > \theta_0$ (15marks)

b. Define a consistent test (2 marks)

c. Consider $x_1, x_2, \ldots, x_n$ of iid continuous random variables with mean μ and known variance σ^2 . We wish to test the hypothesis$H_0: \mu = \mu_0$ $against$ $H_0: \mu < \mu_1$. Show that the test is consistent (3marks)

Question THREE

a. Let $x_1, x_2,..., x_n$ be independently identically distributed $N(\mu, \sigma^2)$random variables where σ^2 is unknown. Find a size α likelihood ratio test for testing $H_0; \mu = \mu_0$ against $H_1; \mu \neq \mu_0$ (10 marks)

b. State and prove the Neyman Pearson lemma (10marks)

Question FOUR

a. Let $x_1, x_2,..., x_n$ be independently identically distributed $N(\mu, \sigma^2)$random variables, Let $y_1, y_2,..., y_n$ be independently identically distributed $N(\mu, \sigma^2)$random variables. Where σ^2 is common. Suppose $X'_i\, s$ and $Y'_i\, s$ are independent. Determine a size α LRT test for $H_0; \mu = \mu_0$ against $H_1; \mu \neq \mu_0$
(12 marks)

b. A company has developed a new hair shampoo named Brightglow. The company marketing executive has obtained figures for the costs of the plant expansion and new product advertising. Taking these costs into account, the executive thinks it would be a mistake to market Brightglow unless there is substantial evidence that more than 20% of the shampoo buyers would choose Brightglow rather than a competitive shampoo. The executive wants the chance of marketing Brightglow to be 0.01 (quite low) if it does not have more than 20% of the market. The plan is to stock a random sample of stores with Brightglow and have a random sample of 500 customers observed as they select a shampoo to purchase. Perform a hypothesis test if 110 of the customers in the sample purchased the Brightglow. (8marks)

Question FIVE

a. Let $x_{i1}, x_{i2}, \ldots, x_{in}$ be independent normally distributed random variables with mean μ_i and variance σ_i^2. Determine a α likelihood ratio test for the hypothesis of the form $H_0; \sigma_i^2 = \sigma_j^2$ against $H_1; \sigma_i^2 \neq \sigma_j^2$ (13 marks)

b. Obtain the power function of the test developed in (a) above (7marks)