TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 5106: TEST OF HYPOTHESIS

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2019

TIME: 3 HOURS

DATE: AUGUST 2019

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five questions. Attempt QUESTION ONE and any other TWO.

Do not write on the question paper.

Question ONE

a. Define a uniform most powerful test (4 marks)

b. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test $H_0; \mu = 5$ Volts against $H_1; \mu \neq 5$ Volts, using 8 units.

   i. The acceptance region is $4.85 \leq \bar{x} \leq 5.15$ Find the size of the test. (4marks)

   ii. Find the power of the test for detecting a true mean output voltage of 5.1 Volts. (5marks)

c. Let $x_1, x_2, \ldots, x_n$ be a random sample from $f(x;\theta) = \theta e^{-\theta x}$ where $\theta = \theta_0$ and $\theta = \theta_1$ $(\theta_1 > \theta_0)$. Obtain the uniformly most powerful test for $H_0: \theta = \theta_0 \; against \; H_0: \theta < \theta_1$. (7marks)

d. Define the power function of a test (4marks)

e. The capacities of brand 1 and brand 2 window 2 window air conditioners are rated the same. Capacities of random samples of six units of each brand were determined. The sample data is given below. Perform a hypothesis test at the 5% level of significance to determine whether mean consumptions for the brands differ. (6marks)

Brand 1	6.1	6.4	5.6	6.2	6.4	5.9
Brand 2	4.8	5.2	5.3	5.1	5	5.2

Question TWO

a. Let $x_1, x_2, \dots, x_n$ be iid normally distributed random variable with mean μ and known variance σ^2. Develop a likelihood ratio test for testing the hypothesis $H_0: \mu = \mu_0 \; against \; H_0: \mu = \mu_1$ at 5% level of significance (10 marks)

b. Let $x_1, x_2, \dots, x_n$ be a random sample of size 25 from a normal distribution with unknown mean μ and standard deviation 1. Consider the hypothesis $H_0: \mu = 2 \; against \; H_0: \mu = 4$ at 5% obtain a test that maximizes the power when $\mu = 4$ (10marks)

Question THREE

a. Define an unbiased test (5 marks)

b. Consider $x_1, x_2, \dots, x_n$ of iid continuous random variables with mean μ and known variance σ^2 . We wish to test the hypothesis$H_0: \mu = \mu_0 \; against \; H_0: \mu < \mu_1$. Show that the test is unbiased (5marks)

c. Prove that every most powerful or uniformly most powerful criteria region is necessarily unbiased (10marks)

Question FOUR

a. Show that if sufficient test exists there exists a most powerful critical region (10marks)

b. Random samples of 500 men and 500 women have been selected to determine whether the proportion of women favoring a political candidate is greater than the proportion of men favoring the candidate. Carry out a hypothesis test at the 1% level if 40 women and 20 men favor the candidate (10marks)

Question FIVE

a. Define the likelihood ratio test (5marks)

b. Let $x_{i1}, x_{i2}, \dots, x_{in}$ be independently identically distributed $N(\mu_i, \sigma_i^2)$ random variables for $i = 1,2,\dots, k$. Find a size α LRT test for $H_0; \mu_i = \mu_j$ against $H_1; \mu_i \neq \mu_j$ (15 marks)