TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

MASTER OF SCIENCE IN APPLIED STATISTICS

AMA 5108: APPLIED TIME SERIES

END OF SEMESTER EXAMINATION

SERIES: *AUGUST* **TIME:** ***3*** HOURS

DATE: *AUGUST 2019*

INSTRUCTIONS:
ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS FROM THE REMAINING QUESTIONS

QUESTION ONE [30 MARKS]

a) How would you distinguish between MA(1) and AR (1) processes [4 marks]

b) Consider the MA (2) process $Y_t = \mu + e_t - 1.3e_{t-1} + 0.6e_{t-2}$ where e_t are white noise process with mean zero and $\sigma^2 = 1.8$. Find the variance, auto covariance function and the ACF of Y_t [6 marks]

c) Discuss the different components of a time series [8 marks]

i. Define a positive definite function for $x \in X$ [2marks]

ii. Show that the covariance function of the stationery time series {X_t} is positive definite [4 marks]

d) Let {X_t} be a moving average process of order 2 given by $X_t = e_t + \theta e_{t-2}$ where

$\{e_t\}$ is WN (0, 1). Compute the variance of the sample mean $\frac{X_1 + X_2 + X_3 + X_4}{4}$ when $\theta = 0.8$ and when [6marks]

QUESTION TWO [20 MARKS]

Consider the AR(1) process $X_t = \alpha X_{t-1} + e_t$ for $|\alpha| < 1$.

a) By successive replacements of the X_t's in the process show that the process converges to an infinite moving average process of white noise. Hence show that the autocorrelation function for this process is given by $r(h) = \alpha^h$ [10 marks]

b) Let $X_t = \frac{1}{4} X_{t-1} + e_t$. Obtain the spectral density for this process [10 marks]

QUESTION THREE [20 MARKS]

a) Consider the AR (2) process given by $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + e_t$. By obtaining the corresponding Yule walker equations, show that the autocorrelation function when we have distinct roots is given by

$\rho(h) = \left(\frac{\alpha_1 - \pi_2(1-\alpha_2)}{(1-\alpha_2)(\pi_1 - \pi_2}\right)\pi_1^h + \left(\frac{\pi_1 - \alpha_1 - \alpha_2\pi_1)}{(1-\alpha_2)(\pi_1 - \pi_2}\right)\pi_2^h$ [9 marks]

b) Consider the process given by $X_t = X_{t-1} - \frac{5}{16} X_{t-2} + e_t$

i) Is the process stationary [4 marks]

ii) Find its A.C.F [7 marks]

QUESTION FOUR [20 MARKS]

Consider the $2m + 1$ point moving average linear filter given $Y_t = \frac{1}{2m+1} \sum_{j=-\infty}^{\infty} X_{t-j}$.

Let $X_t = e^{i\lambda t} = cos\lambda t + isin\lambda t$

a) Show that Y_t can be written as $Y_t = \frac{1}{2m+1} \frac{\sin\left(\frac{2m+1}{2}\right)\lambda\, exp(i\lambda t)}{sin\left(\lambda/2\right)}$ [10 marks]

b) What will be the effect on the trend when [5 marks]

i. $\lambda = 0$

ii. $\lambda \neq 0$ [5 marks]

QUESTION FIVE [20 MARKS]

a) Consider the following data set

t	1	2	3	4	5	6	7	8	9	10
X(t)	97	110	121	117	79	140	75	127	90	119

i) Find the mean and the sample variance of this process [4 marks]

ii) Find the autocorrelation at lag 1 and Lag 2 [8 marks]

b) Let a time series be given by $X_t = X_{t-1} + e_t$ where e_t is white noise with mean μ and variance σ^2. Let $X_0 = 0$. Show that X_t is non-stationary [3 marks]

c) Show that the Autocorrelation function of ARMA (1,1) model given by $X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$ is given by $\rho(1) = \frac{(1+\alpha\beta)(\alpha+\beta)}{1+\beta^2+2\alpha\beta}$ [5 marks]