TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

MASTER OF SCIENCE IN APPLIED STATISTICS

AMA 5108: APPLIED TIME SERIES

END OF SEMESTER EXAMINATION

SERIES: *AUGUST* **TIME:** ***3*** HOURS

DATE: *AUGUST 2019*

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO FROM THE REMAINING QUESTIONS

QUESTION ONE [30 MARKS]

a) Define a positive definite function for $x \in X$ [2marks]

b) Consider the process given by $X_t = -0.2X_{t-1} + 0.48X_{t-2} + e_t$.

i) Is the process stationary [5 marks]

ii) Find its A.C.F [7 marks]

c) Let e_1 and e_2 be independent random variables with mean 0 and variance $Var(e_1) = Var(e_2) = \sigma^2$.Let λ_0 be a constant $0 < \lambda_0 < \pi$, and define Y_t by $Y_t = e_1 \cos(\lambda_0 t) + e_t \sin(\lambda_0 t)$. Show that Y_t is stationary and compute the auto covariance and autocorrelation functions. [6 marks]

d) Obtain the autocorrelation function of an AR(2) process whose auxiliary equation has complex roots [10 marks]

QUESTION TWO [20 MARKS]

a) How would you distinguish between MA(1) and AR (1) processes [2 marks]

b) Show that the covariance function of the stationery time series {X$_t$} is positive definite [4 marks]

c) Let {X$_t$} be a moving average process of order 2 given by $X_t = e_t + \theta e_{t-2}$ where $\{e_t\}$ is WN (0, 1). Compute the variance of the sample mean $\frac{X_1 + X_2 + X_3 + X_4}{4}$ when $\theta = 0.8$ and when [6marks]

d) Let $= X_t = e^{i\lambda t} = \cos \lambda t + i \sin \lambda t$. Let $a_j = \frac{1}{2m+1}$, $\forall j \in [-m, m]$. Show that

$$Y_t = \sum_{j=-m}^{m} a_j X_{t-j} \text{ can be expressed as } Y_t = \frac{1}{2m+1}\frac{\sin\left(\frac{2m+1}{2}\right)\lambda}{\sin\left(\frac{\lambda}{2}\right)} e^{i\lambda t}$$ [8marks]

QUESTION THREE [20 MARKS]

a) Consider the MA (2) process $Y_t = \mu + e_t - 1.3e_{t-1} + 0.6e_{t-2}$ where e_t are white noise process with $\sigma^2 = 1.8$. Find the variance, autocovariance function and the ACF of Y_t [14 marks]

b) Suppose that Y_t is a stationary process. Show that the process of the first differences $\{W_t\}$ given by $W_t = Y_t - Y_{t-1}$ is also a stationary process [6 marks]

QUESTION FOUR [20 MARKS]

Consider the AR(1) process $X_t = \alpha X_{t-1} + e_t$ for $|\alpha| < 1$.

a) By successive replacements of the X_t's in the process show that the process converges to an infinite moving average process of white noise. [11marks]

b) Show that the autocorrelation function for this process is given by $r(h) = \alpha^h$ [9 marks]

QUESTION FIVE [20 MARKS]

a) Figure 1 and 2 below shows the ACF and PACF of some time series models. Identify the models. Give reasons for your answer. [6 marks]

FIGURE 1

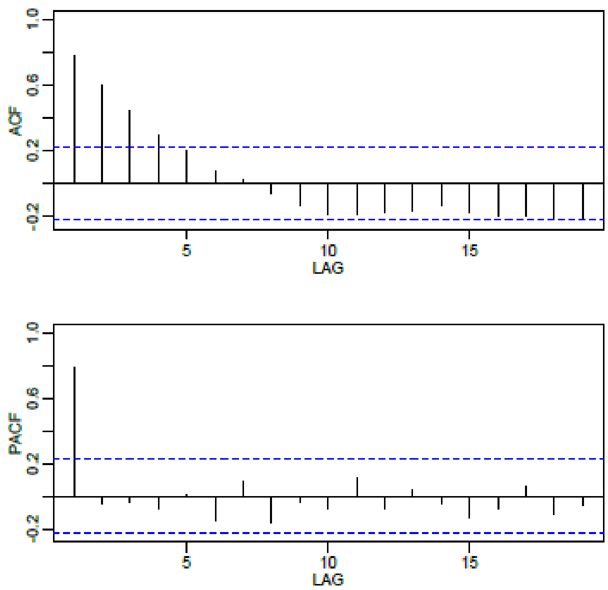
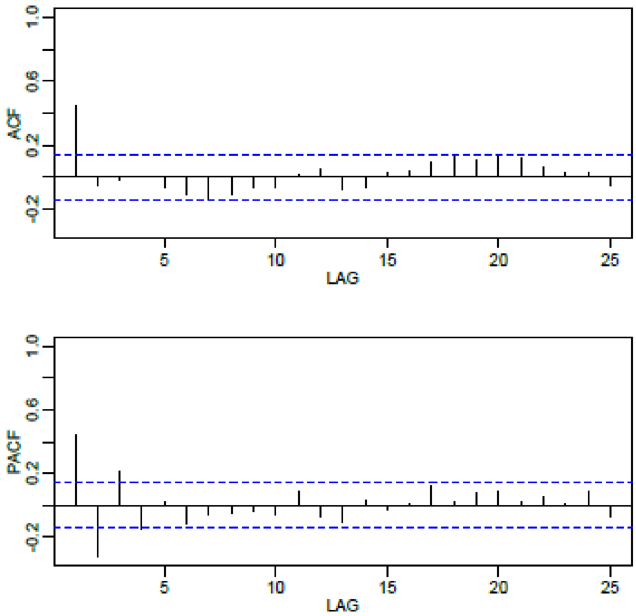


FIGURE 2



b) Explain what the following commands will achieve in the analysis of time series [6 marks]

```
x<-NULL
x[1]<-0
for (i in 2:1000){
  x[i]<-x[i-1]+rnorm(1)
}
print(x)

rw<-ts(x)


plot(diff(rw))

acf(diff(rw))
```

(c) Show that for an MA(1) process, the lag 1 autocorrelation $\rho(1)$ satisfies $-\frac{1}{2} \leq \rho(1) \leq \frac{1}{2}$ [4 marks]

d) For the ARMA(1,2) model $Y(t) = 0.8Y_{t-1} + e_t + 0.7e_{t-1} + 0.6e_{t-2}$ show that

i) $\rho(k) = 0.8\rho(k-1)\,, k \geq 3$

ii) $\rho(2) = 0.8\rho(1) + 0.6\frac{\sigma_e^{\,2}}{\gamma_o}$ [4 marks]