

TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

MASTER OF SCIENCE IN APPLIED STATISTICS

AMA 5108: APPLIED TIME SERIES

END OF SEMESTER EXAMINATION

SERIES: *AUGUST* **TIME:** *3* HOURS

DATE: AUGUST 2019

INSTRUCTIONS:

ANSWER **QUESTION ONE (COMPULSORY)** AND **ANY OTHER TWO** QUESTIONS FROM THE REMAINING QUESTIONS

QUESTION ONE [30 MARKS]

- a) How would you distinguish between MA(1) and AR (1) processes [4 marks]
- b) Consider the MA (2) process $Y_t = \mu + e_t 1.3e_{t-1} + 0.6e_{t-2}$ where e_t are white

noise process with mean zero and $\sigma^2 = 1.8$. Find the variance, auto

covariance function and the ACF of Y_t [6 marks]

c) Discuss the different components of a time series [8 marks]

- i. Define a positive definite function for $x \in X$ [2marks]
- ii. Show that the covariance function of the stationery time series $\{X_t\}$ is positive definite [4 marks]
- d) Let {X_t} be a moving average process of order 2 given by $X_t = e_t + \theta e_{t-2}$ where

 $\{e_t\}$ is WN (0, 1). Compute the variance of the sample mean $\frac{X_1 + X_2 + X_3 + X_4}{4}$ when $\theta = 0.8$ and when [6marks]

QUESTION TWO [20 MARKS]

Consider the AR(1) process $X_t = \alpha X_{t-1} + e_t$ for $|\alpha| < 1$.

- a) By successive replacements of the X_i 's in the process show that the process converges to an infinite moving average process of white noise. Hence show that the autocorrelation function for this process is given by $r(h) = \alpha^h$ [10 marks]
- b) Let $X_t = \frac{1}{4}X_{t-1} + e_t$. Obtain the spectral density for this process [10 marks]

QUESTION THREE [20 MARKS]

a) Consider the AR (2) process given by $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + e_t$. By obtaining the corresponding Yule walker equations, show that the autocorrelation function when we have distinct roots is given by

$$\rho(h) = \left(\frac{\alpha_1 - \pi_2(1 - \alpha_2)}{(1 - \alpha_2)(\pi_1 - \pi_2)}\right) \pi_1^h + \left(\frac{\pi_1 - \alpha_1 - \alpha_2 \pi_1}{(1 - \alpha_2)(\pi_1 - \pi_2)}\right) \pi_2^h$$
 [9 marks]

b) Consider the process given by $X_t = X_{t-1} - \frac{5}{16}X_{t-2} + e_t$

- i) Is the process stationary [4 marks]
- ii) Find its A.C.F [7 marks]

QUESTION FOUR [20 MARKS]

Consider the 2m + 1 point moving average linear filter given $Y_t = \frac{1}{2m+1} \sum_{j=-\infty}^{\infty} X_{t-j}$. Let $X_t = e^{i\lambda t} = cos\lambda t + isin\lambda t$

a) Show that Y_t can be written as $Y_t = \frac{1}{2m+1} \frac{\sin(\frac{2m+1}{2})\lambda \exp(i\lambda t)}{\sin(\lambda/2)}$ [10 marks]

- b) What will be the effect on the trend when [5 marks] i. $\lambda = 0$
 - ii. $\lambda \neq 0$ [5 marks]

QUESTION FIVE [20 MARKS]

a) Consider the following data set

t	1	2	3	4	5	6	7	8	9	10
X(t)	97	110	121	117	79	140	75	127	90	119

i)	Find the mean and the sample variance of this process	[4 marks]
ii)	Find the autocorrelation at lag 1 and Lag 2	[8 marks]

- b) Let a time series be given by $X_t = X_{t-1} + e_t$ where e_t is white noise with mean μ and variance σ^2 . Let $X_0 = 0$. Show that X_t is non-stationary [3 marks]
- c) Show that the Autocorrelation function of ARMA (1,1) model given by $X_t = \alpha X_{t-1} + e_t + e_t$

$$\beta e_{t-1}$$
 is given by $\rho(1) = \frac{(1+\alpha\beta)(\alpha+\beta)}{1+\beta^2+2\alpha\beta}$ [5 marks]