TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 4435: MEASURE, INTEGRATION AND PROBABILITY

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2 HOURS

DATE: September 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of five questions. Attempt Question and any other two Questions.

Do not write on the question paper.

Question ONE (30marks)

a. Show that $X_n \xrightarrow{p} 0$ if $\mathrm{E}|\mathrm{X_n}|^r \to 0$ (4marks)

b. Let $A_n = \left\{\omega; 4 - \frac{2}{3n} < \omega < 8 - \frac{1}{2n}\right\}$ determine if the this sequence is monotone increasing or decreasing hence determine the limit (4marks)

c. Let $A = \{a, b, c, d\}$ determine the power set of A (5 marks)

d. Two dice are tossed and their sums noted. Let X denote the sum of the appearing pair of numbers. Determine the probability distribution of X (5marks)

e. A coin is tossed three times. If X denotes the number of tails and let $Y = \begin{cases} 1 \text{ if } \mathrm{X} \le 1 \\ 2 \text{ if } \mathrm{X} = 2 \\ 3 \text{ if } \mathrm{X} = 3 \end{cases}$ determine the σ field induced by Y (5marks)

f. Define an indicator function (3marks)

g. Show that all fields contain the universal set U (4marks)

Question TWO (20marks)

a. Define the following terms
   i. Probability (3marks)
   ii. Conditional probability measure (3marks)

b. A fair coin is tossed four times. Let X denote the number of tails appearing. Determine;
   i. The sample space (5 marks)
   ii. The distribution function of X (5marks)
   iii. The expectation of X (4marks)

Question THREE (20marks)

a. State and prove Fatou's theorem (12 marks)

b. Define the term independence hence show that if A and B are independent then A and B^c are also independent (8 marks)

Question FOUR (20marks)

a. Show that convergence in probability implies convergence in distribution (10 marks)

b. Define convergence in the r^{th} mean hence show that

   $X_n \xrightarrow{r} X$ implies that $\mathrm{E}|\mathrm{X}_\mathrm{n}|^r \to \mathrm{E}|\mathrm{X}|^r$ (10marks)

Question FIVE(20marks)

a. Show that if $\mathrm{Q(t)}$ is the characteristic function X ,then $\mathrm{Q(t)}$ is continuous (8 marks)

b. State and prove Borel Cantelli lemma (12marks)