TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

. **BACHELOR OF SCIENCE AND COMPUTER SCIENCE**

AMA 4420: DIFFERENTIAL GEOMETRY

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME: 2 HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of Choose No questions. Attempt QUESTION ONE AND ANY OTHER TWO QUESTIONS
Do not write on the question paper.

Question ONE

a) Find the constant a such that the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}-a\hat{j}+5\hat{k}$ are coplanar. (5mks)

b) Find the equation of a plane passing the point $(3,-1,-2)$ and perpendicular to the vector $6\vec{i}+5\vec{j}-8\vec{k}$ (5mks)

c) Determine the equation of the tangent line to the curve $\vec{r}=e^{t}\vec{i}-e^{-t}\vec{j}+t^{2}\vec{k}$ at $t=1$ (5mks)

d) Find the length of the arc $\vec{r}=e^{t}\cos t\vec{e}_1+e^{t}\vec{e}_2+e^{t}\vec{e}_3, 0\le t\le\pi$ (5mks)

e) Find the first fundamental magnitude for surface of revolution $x=f(u)\cos v, y=f(u)\sin v, z=\varphi(u)$ (5mks)

f) Find the curvature of the helix $\vec{r}(t)=a\cos\omega t\hat{i}+a\sin\omega t\hat{j}+bt\hat{k}$ (5mks)

Question TWO

a) If $\vec{A} = \hat{i} + \hat{j}, \vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}, \vec{C} = 4\hat{j} - 3\hat{k}$ find $\vec{A} \times (\vec{B} \times \vec{C})$ (4mks)

b) Find the volume of a parallelepiped with sides $\vec{A} = 3\hat{i} - \hat{j}, \vec{B} = \hat{j} + 2\hat{k}, \vec{C} = \hat{i} + 5\hat{j} + 4\hat{k}$ (4mks)

c) For the curve $\vec{X} = a\cos t\hat{i} + a\sin t\hat{j} + bt\hat{k}$ find the

i. Binormal vector (4mks)

ii. Equation of the binormal line (4mks)

iii. Equation of the rectifying plane (4mks)

Question THREE

a) Define a curve C in Euclidean space E^3. Hence show that the curve $\bar{r} = 2\cos\theta - 1.0 \le \theta \le 2\pi$ is a regular representation of a curve. (5mks)

b) Determine the first fundamental form of the surface $\vec{X} = (u+v)\vec{e}_1 + (u-v)\vec{e}_2 + (uv)\vec{e}_3$ (5mks)

c) Given the space curve $\vec{r} = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ at $t = \pi/2$. Obtain the equation of:

i. Principal normal (5mks)

ii. The osculating plane (5mks)

Question FOUR

a) Find the area of a triangle with vertices at $p(2,3,5), Q(4,2,-1), R(3,6,4)$ (4mks)

b) Show that the binormal of the involute $\vec{X}^* = \vec{X} + (c-s)\hat{t}$ of $\vec{X} = \vec{X}(s)$ is $\hat{b}^* = \dfrac{k\hat{b} + \tau\hat{t}}{|(c-s)k|k^*} = k^*\hat{n}^*$ (9mks)

c) Find the arc length of the arc $r = 3a\cos t\vec{e}_1 + 3a\sin t\vec{e}_2 + 4at\vec{e}_3$ from the point of intersection with the plane $z = 0$ to the arbitrary point $m(t)$ (7mks)

Question FIVE

a) Prove that the curvature of the space curve $\vec{r} = \vec{r}(t)$ is given numerically by $k = \dfrac{\dot{\vec{r}} \times \ddot{\vec{r}}}{\left|\dot{\vec{r}}\right|^3}$ (5mks)

b) Show that along the curve $\vec{x} = \vec{x}(s)$ $\dddot{\vec{x}} = -k^2\hat{t} + \dot{k}\,\hat{n} + \tau k\hat{b}$ (5mks)

c) Determine the second fundamental form for the surface $\vec{X} = u\hat{i} + v\hat{j} + \left(u^2 - v^2\right)\hat{k}$ (10mks)