# FACULTY OF APPLIED AND HEALTH SCIENCES <br> DEPARTMENT OF MATHEMATICS \& PHYSICS <br> UNIVERSITY EXAMINATION FOR THE SECOND YEAR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE. <br> AMA 4419: FLUID MECHANICS III <br> SPECIAL/ SUPPLIMENTARY EXAMINATIONS 

SERIES: SEPTEMBER-2018
TIME: 2 HOURS

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of five (5) questions. Attempt Question One and any other Two Questions.
Use SMP four figure mathematical tables and non-programmable electronic calculators.

## Do not write on the question paper.

Question ONE (30 MARKS) compulsory
a) State the Buckingham's $\pi(\mathrm{Pi})$ theorem.
b) The boundary layer thickness at a distance of 1 m from the leading edge of a flat plate kept over zero angle of incidence to the flow direction is 1 mm . The velocity outside the boundary layer is $25 \mathrm{~m} / \mathrm{s}$, determine the boundary layer thickness at a distance of 4 m if it is entirely laminar.
c) In an oil field a well is sunk into a reservoir of a spherical sphere, the oil is coming from oil bearing rocks into the reservoir. Assuming that pressure $P_{1}$ in the rocks is much higher than the pressure $\mathrm{P}_{0}$ in the reservoir and the permeability of the rocks is $k=k_{0} r^{3}$ where r is the radial distance from the centre of the reservoir. Find what would be the radius of the reservoir so that uninterrupted supply of oil at the rate q can be obtained from the well.
d) Define the following non dimensional parameters:
i) Froude number.
1 mark
ii) Weber number.
1 mark
iii) Mach number.
e) A geometrically similar model of an air duct is built to $1 / 50$ scale and tested with water which is 75 times more viscous and 1200 denser than air. When tested under dynamically similar conditions the pressure drop is $400 \mathrm{KN} / \mathrm{m}^{2}$ in the model. Find the corresponding pressure drop in the full scale prototype.
f) In a step bearing of breadth b, with the step centrally positioned.
i) Show that the volumetric flow rate Q through the bearing is given by $Q=\frac{V\left(1+H^{2}\right)}{2\left(1+H^{3}\right)} b h_{1}$ where

$$
H=\frac{h_{1}}{h_{2}} .
$$

ii) The bearing has the following dimensions; $\mathrm{h}_{1}=0.5 \mathrm{~mm}, \mathrm{~h}_{2}=0.25 \mathrm{~mm}, \mathrm{~L}=100 \mathrm{~mm}, \mathrm{~b}=100 \mathrm{~mm}$, while it is used in conjunction with an oil of relative density 0.87 and kinematic viscosity $2 \times 10^{-4} \mathrm{~m}^{2} \mathrm{~s}^{-1}$. The relative velocity between the bearing surfaces is $10 \mathrm{~m} \cdot \mathrm{~s}^{-1}$, determine the volumetric flow rate of oil.

2 marks
iii) Determine the load supported by the step bearing.

## Question TWO (20 MARKS)

a) A model of submarine is scaled down to $1 / 20$ of the prototype and is to be tested in a wind tunnel where free stream pressure is 2.0 MPa , absolute temperature is $50^{\circ} \mathrm{C}$ and the speed of the prototype is $7.72 \mathrm{~m} / \mathrm{s}$. Determine the free stream velocity of air and the ratio of the drag between model and prototype. Assume kinematic viscosity of sea water as $1.4 \times 10^{-6} \mathrm{~m}^{2} / \mathrm{s}$ and viscosity of air as $0.0184 \mathrm{Ns} / \mathrm{m}^{2}$.
b) Show that the mass flux in a porous annulus of radii $r=a$ and $r=b$ where $b>a$ in which the pressure $P=P_{0}$ when $\mathrm{r}=\mathrm{a}$ and $\mathrm{P}=\mathrm{P}_{1}$ when $\mathrm{r}=\mathrm{b}$ if permeability k is a constant will be $q=\frac{2 \pi k}{\mu}\left(\frac{P_{1}-P_{0}}{\ln a / b}\right)$.
c) Obtain an expression for the ratio of displacement thickness to momentum thickness, for a steady Poiseuille flow in a cylindrical pipe of radius R given that the velocity distribution is $u=-\frac{1}{4 \mu} \cdot \frac{\partial p}{\partial x}\left(R^{2}-y^{2}\right)$ where y is the distance measured perpendicularly inside the pipe from the centre line.

## Question THREE (20 MARKS)

a) If the velocity distribution in the boundary is given by $\frac{u}{U}=\left(\frac{y}{\delta}\right)^{1 / 7}, \delta$ being boundary layer thickness.

Calculate: i) Displacement thickness. 2 marks
ii) Momentum thickness. 2 marks
iii) Shape factor. 1 mark
iv) Energy thickness.

3 marks
v) Energy loss due to boundary layer if at a particular section, the boundary layer thickness is 25 mm and the free stream velocity is $15 \mathrm{~m} / \mathrm{s}$.

2 marks
vi) If the discharge through the boundary layer region is $6 \mathrm{~m}^{3} / \mathrm{s}$ per metre width, express this energy loss in terms of metres of head. Take $\rho=1.2 \mathrm{~kg} / \mathrm{m}^{3}$.

1 mark
b). A plane bearing plate is traversed by a wide inclined slipper of 15 cm in length moving at $2 \mathrm{~m} / \mathrm{sec}$. The clearance between the slipper and bearing plate is 0.03 mm at the toe and 0.09 mm at the heel. If the bearing has to carry a load of $30,000 \mathrm{kgf}$ per metre width; i) Determine the viscosity of lubricating oil.
ii) The maximum pressure in the lubricant.
iii) The power consumed per metre width of bearing.

## Question FOUR (20 MARKS)

a) The performance for a spillway of an irrigation project is to be studied by means of a model constructed to a scale of 1:9 neglecting the viscous and surface tension effects determine:
i) Rate of flow in model for a prototype discharge of $1200 \mathrm{~m}^{3} / \mathrm{s}$. 3 marks
ii) The dissipation of energy in the prototype hydraulic jump, if the jump in the model dissipates 0.25 kW .

3 marks
b) The velocity distribution in the boundary over the face of an open channel was observed to be $\frac{u}{U}=\left(\frac{y}{\delta}\right)^{0.22}$, the free stream velocity $U$ is $20 \mathrm{~m} / \mathrm{s}$ and boundary layer thickness 5 cm at a certain section. The discharge $5 \mathrm{~m}^{3} / \mathrm{s}$ per meter length of the open channel. Calculate: i) Displacement thickness.
ii) Energy thickness.
iii) Loss of energy up to the Section under consideration.

3 marks
c) State any four uses of dimensional analysis.

## Question FIVE (20 MARKS)

a) A plate $450 \mathrm{~mm} \times 150 \mathrm{~mm}$ has been placed longitudinally in a stream of crude oil of specific gravity 0.925 and kinematic viscosity of 0.9 stoke, which flows with velocity of $6 \mathrm{~m} / \mathrm{s}$. Calculate
i) Friction drag on the plate.
5 marks
ii) Thickness of the boundary layer at the trailing edge.
2 marks
iii) Shear stress at the trailing edge.
4 marks
b) A 1:40 model of an ocean tanker is dragged through fresh water at $2 \mathrm{~m} / \mathrm{s}$ with a total measured drag of 117.7 N , the skin frictional drag coefficient f for model and prototype are 0.3 and 0.02 respectively in the equation $R_{f}=f A V^{2}$. The wetted surface area of the model is $25 \mathrm{~m}^{2}$, taking densities for the prototype and model as $1030 \mathrm{kgN} / \mathrm{m}^{3}$ and $1000 \mathrm{kgN} / \mathrm{m}^{3}$ respectively. Determine the total drag on the prototype.
c) State Darcy's law and write its formula in the usual symbols.

