TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

SEPTEMBER 2018 SERIES EXAMINATION

UNIT CODE: AMA 4418 UNIT TITLE: ANALYTICAL APPLIED MATHEMATICS II

BMCS

SPECIAL/SUPPLIMENTARY EXAMINATION

TIME ALLOWED: 2 HOURS

INSTRUCTIONTO CANDIDATES:

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a) Given that n = 2, write two equations for $y_i = c_i^r a_{rs} x_s$ (4 marks)

b) If $\bar{A}^p = \frac{\partial \bar{x}^p}{\partial x^q} A^q$ prove that $A^q = \frac{\partial x^q}{\partial \bar{x}^p} \bar{A}^q$ (3 marks)

c) Given that A_r^{pq} and B_r^{pq} are tensors, prove that their sum and difference are tensors.

(4 marks)

d) Define an affine tensor and hence a Cartesian tensor. (2 marks)

e) show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one. (3 marks)

f) Consider the initial value problem;

$$\frac{d^2y}{dx^2} + xy = 1 \,,$$
$$y(0) = 0 \,, y'(0) = 0$$

Transform this initial value problem to a Volterra integral equation. (7 marks)

g) Define a singular integral equation. (2 marks)

h) A quantity A (j,k,l,m) which is a function of coordinates x^i transforms to another coordinate system $\bar{x}^i$ according to the rule.

$$\bar{A}\,(p,q,r,s) = \frac{\partial x^j}{\partial \bar{x}^p}\frac{\partial \bar{x}^q}{\partial x^k}\frac{\partial \bar{x}^r}{\partial x^l}\frac{\partial \bar{x}^s}{\partial x^m}A(j,k,l,m)$$

i) Write the tensor in a suitable notation. (2 marks)

ii) Give the contravariant and covariant order and the rank of the tensor. (3marks)

QUESTION TWO (20 MARKS)

a) A_j^i is a mixed tensor of rank 2 and B_m^{kl} is a mixed tensor of rank 3, prove that $A_j^i B_m^{jl}$ is a mixed tensor of rank 3. (5 marks)

b) Show that the expression A(i,j,k) is a covariant tensor of rank 3 if $A(i,j,k)B^k$ is covariant tensor of rank 2 and B^k is a contravariant vector. (5 marks)

c) Show that the Bessel equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \lambda(x^2 - 1) = 0 \,,$$
$$y(0) = 0 \,, y(1) = 0$$

transforms to the integral equations

$$y(x) = \lambda \int_0^1 G(x,\xi)\xi y(\xi) d\xi$$

Where

$$G(x,\xi) = \begin{cases} \frac{x}{2\xi}(1-\xi^2), x < \xi \\ \frac{\xi}{2x}(1-x^2), x > \xi \end{cases}$$ (10 marks)

QUESTION THREE (20 MARKS)

a) Find the matrix and component of first fundamental tensors in cylindrical coordinates (12 marks)

b) Prove that the Legendres polynomial of order three is given by

$p_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$ (4 marks)

c) *If* A(i,j,k)$A^iB^jC_k$ is a scalar for arbitrary vectors $A^iB^jC_k$ show that A(i,j,k) is a tensor of type (1,2) (4 marks)

QUESTION FOUR (20 MARKS)

a) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0\ ,$$

$$y(0) = 0\ , y(l) = 0$$

Transform this boundary value problem to a Fredholm equation of the second kind.

(8 marks)

b) A curve in spherical coordinates x^i is given by $x^1 = t\ \ x^2 = \sin^{-1}(\frac{1}{t})$ and $x^3 = 2\sqrt{t^2+1}$. find the length of arc $1 \leq t \leq 2$ (8 marks)

c) Express interms of Legendres polynomial the function

f(x) = x^4-$2x^3$+ $3x^2$+5x-9 (4 marks)

QUESTION FIVE (20 MARKS)

a) Given cylindrical coordinate (x^i) and rectangular coordinates $(\bar{x}^i)$ are connectd through $\bar{x}^i = x^1 cosx^2\ \ \bar{x}^2 = x^1 sinx^2\ \ \bar{x}^3 = x^3$ compute the jacobian matrix and thus compute the metric G of the Euclidean metric tensor in the x^i system. (8 marks)

b) Let A^{pq}_{rst} be a tensor

i) Choose p =t and show that A^{pq}_{rst}, where the summation convention is employed is a tensor and state its rank. (4marks)

ii) Choose p= t and q=s and show similarly that A^{pq}_{rqp} is a tensor and state its rank. (3marks)

c) i) Define Legendre polynomials and Legendre functions of second kind. (2marks)

ii) Prove that $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$ (3marks)