AAA

TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BMCS YEAR IV SEMESTER I

AMA 4406: GROUP THEORY II

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2HOURS

DATE: Pick Date**September 2018**

Instructions: Answer question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

a) Define the terms:

i) Composition series (1 mark)

ii) Nilpotent group (1 mark)

iii) Decomposable group (1 mark)

iv) Sylow p-subgroup of a group (1 mark)

v) Maximal normal subgroup of a group. (1 mark)

b) Let H be a Sylow p-subgroup of a finite group G and $N_G(H)$ the normalizer of H in G, then $\frac{N_G(H)}{H}$ has no elements whose order is a power of a prime p except the identity. (4 marks)

c) All finite abelian groups are soluble. Prove. (5 marks)

d) Any two composition series of a group are equivalent. Prove. (3 marks)

e) List all the abelian groups of order 8. (3 marks)

f) List all the subnormal series of $\mathbb{Z}_{15}$ (3 marks)

g) Show that there is no simple group of order 200. (3 marks)

h) Establish whether the quaternion group is nilpotent. (4 marks)

QUESTION TWO (20 MARKS)

a) State any one of the three Sylow theorems. (3 marks)

b) (i) Define a $\boldsymbol{p}$- group. (2 marks)

(ii) Let $n, p, r, m \in \mathbb{Z}^+$ such that $m \nmid p$ for a prime p. Show that a finite group G is a p-group if and only if its order is $n = p^r m$. (5 marks)

(iii)What is the order of a Sylow 3-subgroup of a group of order 54? (2 marks)

c) Find all the Sylow 2- subgroups of S_3 and demonstrate that they are conjugate. (7 marks)

d) Prove that if N is a normal subgroup of G that contains a Sylow p-subgroup of G, then the number of Sylow p-subgroups of N is the same as that of G. (3 marks)

QUESTION THREE (20 MARKS)

a) Find all the Maximal and maximal normal subgroups of the quaternion group. (5 marks)

b) List all the subnormal series of D_4 (5 marks)

c) (i) State Schreier theorem. (2 marks)

(ii) Use $\{0\} \lhd 30\mathbb{Z} \lhd 5\mathbb{Z} \lhd \mathbb{Z}$ and $\{0\} \lhd 6\mathbb{Z} \lhd \mathbb{Z}$ to illustrate Schreier theorem. (5 marks)

d) Use A_5 to show solvability of non-abelian simple groups. (3 marks)

QUESTION FOUR (20 MARKS)

a) Determine whether the quaternion group is soluble. (5 marks)

b) Let G' denote the derived group of a group G. show that G' is a normal subgroup of G and $\frac{G}{G'}$ is abelian. (5 marks)

c) A finite group G is soluble if it contains a normal subgroup K such that K and $\frac{G}{K}$ are soluble. Prove. (5 marks)

d) Show that every nilpotent group is soluble. (5 marks)

QUESTION FIVE (20 MARKS)

a) (i) Let H and K be subgroups of a group G. Define internal direct product of H and K. (2 marks)

(ii) Under multiplication, let $C_2 = \{1, a\}$ and $C_3 = \{1, b, b^2\}$. Find $C_2 \times C_3$ and show that $C_2 \times C_3$ is cyclic. (4 marks)

b) Find all abelian groups of order 360 (up to isomorphism). (6 marks)

c) Let G be an abelian group. Explain what is meant by torsion group T. Hence, show that T in G is a subgroup of G. (4 marks)

d) Let m be a square free integer. Show that every abelian group of order m is cyclic. (4 marks)