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TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BMCS YEAR IV SEMESTER I

AMA 4405: COMPLEX ANALYSIS II

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2HOURS

DATE: Pick Date**September 2018**

Instructions: Answer question ONE and any other TWO questions.

QUESTION ONE (30 MARKS)

(a) (i) Show that the function $f(z)=\frac{e^z-1}{z}$ has a removable singular point. (4 marks)

(ii) Redefine the above function to make it analytic everywhere. (2 marks)

(b) Show that the function $f(z)=e^{\frac{1}{z^2}}$ has an essential singular point. (5 marks)

(c) Show that $f_1(z)=\sum_{n=0}^{\infty} z^n$ and $f_2(z)=\frac{1}{1-z}$ are analytic continuation of each other. (4 marks)

(d) (i) Let $f(z)=\frac{\phi(z)}{(z-z_0)^m}$, m=2,3,..... by the use of Taylor's series expansion, derive the residue of the function. (4 marks)

(ii) Hence find the residue of $f(z)=\dfrac{z^3+2z}{(z-i)^3}$ (4 marks)

(e) Evaluate $\int_{-\infty}^{\infty}\dfrac{dx}{(x^2+1)(x^2+4)}$ (7 marks)

SECTION B

QUESTION TWO (20 MARKS)

(a) Define a conformal mapping. (2 marks)

(b) Consider the curves $L_1: x=2$ and $L_2: y=2$ meeting at $z_0=(2,2)$. Investigate conformity of $T(z)=z^2$ at $z_0=(2,2)$. (8 marks)

(c) Find the fixed point of the transformation $T(z)=\dfrac{4iz+5}{z+2i}$ (4 marks)

(d) Find a bilinear transformation that maps $z_1=1, z_2=0, z_3=-1$ onto $w_1=i$, $w_2=\infty$ and $w_3=1$ respectively. (6 marks)

QUESTION THREE (20 MARKS)

(a) (i) State the Schwartz Reflection Principle (2 marks)

(ii) Show that $f(z)=2z+3$ satisfies the Schwartz Reflection Principle (2 marks)

(b) (i) Define an infinite product (3 marks)

(ii) Investigate the convergence of the product $\left(1-\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\left(1-\dfrac{1}{4}\right).....$ (3 marks)

(iii) Investigate the convergence of the product $\prod_{n=1}^{\infty}\left(1-\dfrac{1}{n+1}\right)$ (4 marks)

(c) Determine a Mobius transformation that maps $z_1=i, z_2=0, z_3=1$ onto $w_1=1, w_2=\infty, w_3=-i$. (6 marks)

QUESTION FOUR (20 MARKS)

(a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ (6 marks)

(b) By the Residue Theorem, evaluate $\int_c \frac{5z-2}{z(z-1)}$,c:$|z|=2$ using the two singular points of the function (10marks)

(c) Expand the given function into its partial fraction representation using the theory of residues. $f(z)=\frac{z^2-7z+4}{z^2(z+4)}$ (4 marks)

QUESTION FIVE (20 MARKS)

(a) Consider the triangle ABC in the plane with A(0,1), B(1,1) and C(1,0). Find $T(z)=z^2$. Discuss the conformality of T at C. (6 marks)

(b) Given that $U(x,y)=xy$ is harmonic, find the harmonic conjugate V of U and write the resulting analytic function in terms of complex variable z. (6 marks)

(c) (i) Define an isolated singular point $z=z_o$. (2 marks)

(ii) By using a single residue at $z=0$, evaluate $\int_c \frac{5z-2}{z(z-1)}$,c:$|z|=2$ (6 marks)