TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS / BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY

AMA 4401: COMPLEX ANALYSIS

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2 HOURS

DATE: September 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of FIVE questions. Attempt QUESTION ONE and ANY OTHER TWO QUESTIONS

Do not write on the question paper.

QUESTION ONE (30 Marks)

a) Find the roots of $z^5 = -32$ [6 Marks]

b) Evaluate $\lim_{z \to -i} \frac{z+i}{z^2+1}$ [3 Marks]

c) Discus the continuity of $f(z) = \frac{3z^4 + 2z^3 + 8z^2 + 2z + 5}{z+i}$ [4 Marks]

d) For the arc C and the function f, find the value of $\oint_C f(z)dz$ given that C is a contour and f is continuous on C if $f(z) = \frac{z+2}{z}$ and C is the semicircle $z = 2e^{i\theta}$ for $\pi \le \theta \le 2\pi$ [6 Marks]

e) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^3} dz \quad C: |z| = 1$ [5 Marks]

f) Evaluate $\int_C \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$

[6 Marks]

QUESTION TWO (20 Marks)

a) Find the singularities and the corresponding residues of the function $f(z) = \dfrac{e^z}{z^2(z^2 + 2z + 2)}$ [11 Marks]

b) Use residues to evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ [9 Marks]

QUESTION THREE (20 Marks)

a) Evaluate $(7 + 2i\sqrt{3})(5 - 4i\sqrt{3})$ [2 Marks]

b) Show that the multiplicative inverse of the complex number $z = (x, y)$ is $\left(\dfrac{x}{x^2 + y^2}, \dfrac{-y}{x^2 + y^2}\right)$. Hence or otherwise find the inverse of $z = 3 - 4i$ [10 Marks]

c) Solve for the real values of x and y in the equation $\left(\dfrac{1+i}{1-i}\right)^2 + \dfrac{1}{x + iy} = 1 + i$ [8Marks]

QUESTION FOUR (20 Marks)

a) Show that under the transformation $w = \dfrac{1}{z}$, the images of the lines $y = x - 1$ and $y = 0$ are the circles $u^2 + v^2 - u - v = 0$ and $v = 0$ respectively. Sketch the two pairs of curves and verify the conformality of the mapping at $z = 1$ [12 Marks]

b) Find the Laurent series of $\dfrac{z}{(z+1)(z+2)}$ about $z = -2$ [8 Marks]

QUESTION FIVE (20 Marks)

a) Show by De Moivre's theorem that $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$ [8 Marks]

b) Suppose that $z = a\cos\omega t + bi\sin\omega t$ (where a, b, ω are positive constants, $a > b$) is the position vector of a particle moving on a curve C and that t is the time.

   i. Determine the velocity and speed of the particle at any time [2 Marks]

ii. Determine the acceleration both in magnitude and direction at any time. [2 Marks]

iii. Prove that $\frac{d^2z}{dt^2} = -\omega^2 z$ and give a physical interpretation [3 Marks]

iv. Determine where the velocity and acceleration have the greatest and least magnitudes. [6 Marks]