TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

SERIES: September 2018

AMA 4321: ANALYTICAL APPLIED MATHEMATICS 1

BMCS

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. (i) Define a linear second order differential equation. (2 mk)

(ii) Solve the differential equation using D operator method (7 mks)

b. Determine the Laplace transform of $sin^2 t$ (4 mks)

c. Determine the Legendre polynomial using the solution of Legendre equation given by

$y = a_0 \left\{1 - \frac{k(k+1)}{2!}x^2 + \frac{k(k+1)(k-2)(k+3)}{4!}x^4 \dots\dots\right\} + a_1 \left\{x - \frac{(k-1)(k+2)}{3!}x^3 + \frac{(k-1)(k-3)(k+2)(k+4)}{5!}x^5 \dots\dots\right\}$ (6 mks)

d. Verify that $\emptyset(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ satisfies the partial differential equation $\frac{\partial^2\emptyset}{\partial x^2} + \frac{\partial^2\emptyset}{\partial y^2} + \frac{\partial^2\emptyset}{\partial z^2} = 0$ (6 mks)

e. Obtain a Fourier series for the periodic function f(x) defined as $f(x) = \begin{cases} -k, -\pi < x < 0 \\ +k, \ 0 < x < \pi \end{cases}$ (5 mks)

QUESTION TWO (20 MARKS)

a. Find (i) $\frac{\partial^2 z}{\partial y^2}$ (2 mks)

(ii) $\frac{\partial^2}{\partial x}\frac{z}{\partial y}$ Given $z = 4x^2y^3 + 7y^2 + 2$ (2 mks)

b. A metal bar insulated along its sides is 1 meter long. It is initially at room temperature of 15^0c and at time t=0, the ends are placed into ice at 0^0c. Find an expression for the temperature at a point p at a distance x meters from one end at any time t seconds after t=0. (7 mks)

c. Determine the Legendre polynomial $p_3(x)$ using the Rodrigue's formula

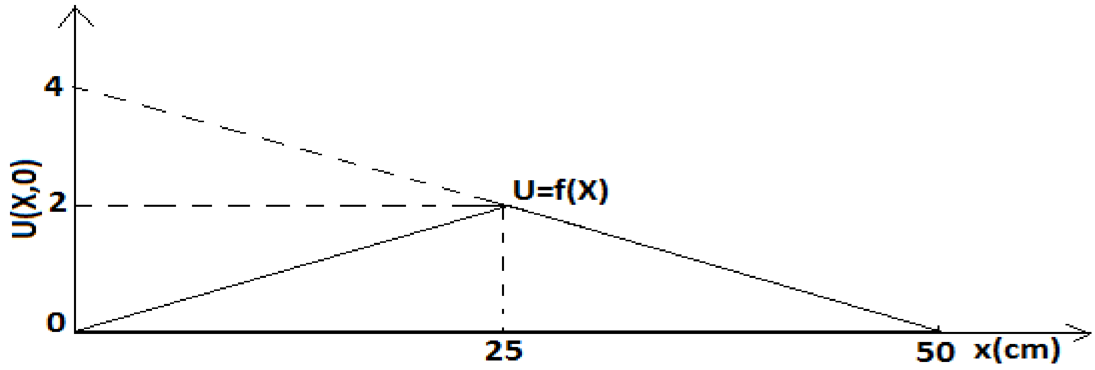
d. Solve the partial differential equation by direct partial integration (5 mks)

$\frac{\partial^2}{\partial x}\frac{u}{\partial y} = \cos(x + y)$ given $\frac{\partial y}{\partial x} = 2 \ when \ y = 0 \ and \ u = y^2 \ when \ x = 0$ (4 mks)

QUESTION THREE (20 MARKS)

a. In the figure below is a stretched string of length 50 cm which is set oscillating by displacing its midpoint a distance of 2 cm from its rest position and releasing it with zero velocity. Solve the formed equation by method of separating the variables.

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ Where $c^2 = 1$, to determine the resulting motion $u(x, t)$ (10 mks)



b. Find the power series solution of the Bessel equation given by

$x^2\frac{dy}{dx}+x\frac{dy}{dx}+(x^2-v^2)y=0$ in terms of $J_v(x) and\ J_{-v}(x)$ (10 mks)

QUESTION FOUR (20 MARKS)

a. (i) Determine the Fourier Series for the function $f(\theta)=\theta^2$ in the range $-\pi<\theta<\pi$. The function has a period of 2π. (2 mks)

(ii) If $\theta=\pi$, show that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ (6 mks)

b. Solve the partial differential equation by direct integration method

$\frac{\partial^2 u}{\partial x^2}=6x^2(2y-1)$ Given the boundary conditions that at $x=0$ $\frac{\partial u}{\partial x}=sin2y$ and $u=\cos y$ (6 mks)

c. Solve the given second order linear differential equation

$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2Y=3e^x\cos 2x$ Given that when $x=0$ $y=2\ and\ \frac{dy}{dx}=3$ (6 mks)

QUESTION FIVE (20 MARKS)

a. A square plate is bounded by the lines $x=0, y=0, x=1\ and\ y=1$. apply the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ to determine the potential distribution u(x,y) over the plate subject to the boundary conditions.

$U=0$ when $x=0$ $0\le y\le 1$

$U=0$ when $x=1$ $0\le y\le 1$

$U=0$ when $y=0$ $0\le x\le 1$

$U=4$ when $y=0$ $0\le x\le 1$ (10 mks)

b. (i) Find the Laplace Transform of $3\sin(\omega t+\propto)$. $where\ \omega\ and\ \propto are\ constants$ (3 mks)

(ii) use Laplace Transform to solve the differential equation

$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+13y=0$ given that when $x=0$ $y=3\ and\ \frac{dy}{dx}=7$ (7 mks)