



TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4316: ORDINARY DIFFERENTIAL EQUATIONS

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME: 2 HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of FIVE questions. Attempt QUESTION ONE and any other TWO questions.

Do not write on the question paper.

QUESTION ONE Compulsory (30 marks)

- a) Explain the following terms;
- Ordinary differential equation. (2 marks)
 - Degree of a differential equation. (2 marks)
- b) Solve the initial value problem $\frac{dy}{dx} = -4xy^2, y(0) = 1$ (4 marks)
- c) The population growth rate of TUM is 3%. Find the population N after time t if initially the population was n_0 (4 marks)
- d) Find the General Solution. of $y'' + y' + y = 0$ (3 marks)
- e) Determine the function $F(t)$ for which $L\{F(t)\} = \frac{3}{5} - \frac{4e^{-s}}{s^2} + \frac{5e^{-2s}}{s^2}$ (4 marks)
- f) Use the method of undetermined coefficient to find the general solution of $\frac{d^2y}{dx^2} + \frac{14dy}{dx} + 49y = 4e^{5x}$ (6 marks)
- g) Show that $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ is exact hence solve it. (5 marks)

QUESTION TWO (20 marks)

- a) Eliminate the constants from $e^x(A \cos X + B \sin X)$ and obtain the differential equation. (5 marks)
- b) Find :
- i) A particular solution of $\sin X \frac{dy}{dx} + y \cos x = x \sin x, y\left(\frac{\pi}{2}\right) = 2$ (7 marks)
- ii) The general solution of $\frac{d^2 y}{dx^2} - \frac{6dy}{dx} + 9y = 0$ (4 marks)
- c) Solve the initial value problem $\frac{dy}{dx} = 3x^2 + 4x + 2$ subject to $y(0) = -1$ (4 marks)

QUESTION THREE (20 marks)

- a) A radioactive substance decays to half its original mass after half-life. Find a relation for the half-life $\frac{t}{2}$ and the decay substance λ (7 marks)
- b) Solve the following equation using the method indicated
- i) $5 \frac{dy}{dx} + 2x = 3, y(2) = \frac{7}{5}$ [direct integration] (4 marks)
- ii) $y'' - 3y' - 4y = xe^{2x}$ [undetermined coefficient] (9 marks)

QUESTION FOUR (20 Marks)

- a) A function $F(t)$ is defined by $F(t) = 4 \quad 0 < t < 2$
 $= 2t - 3 \quad \text{for } 2 < t$ Sketch the graph of the function and determine its Laplace transform. (6 marks)
- b) Solve the Bernoulli's equation $xy(1 + xy^2) \frac{dy}{dx} = 1$ (9 marks)
- c) The slope m of a curve is given by $m = \frac{y+3}{x+2}$. If the curve passes through the point $\left(\frac{1}{2}, 1\right)$ find its equation. (5 marks)

QUESTION FIVE (20 marks)

- a) Reduce
- i) $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$ to homogenous form, hence (5 marks)
- ii) Solve (i) above (8 marks)
- b) Solve $x' = y$
 $y' = -2x + 3y$ (7 marks)