TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

UNIVERSITY EXAMINATION FOR THE FIRST SEMESTER IN THE SECOND YEAR OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

AMA4314: REAL ANALYSIS I

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME: 2HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

QUESTION ONE (30 MARKS)

(a) Given a sequence $\{x_n\}$ where $x_n = 1 + \frac{1}{n}$, find the least positive integer N such that $|x_n - 1| < \varepsilon$ for all $n > N$ if $\varepsilon = 0.001$. (3mks)

(b) Find the closure of $A_n = \left\{(-1)^n \frac{1}{n+1} : n \in \mathbb{N}\right\}$. (3mks)

(c) Show that the set $S = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$ is bounded. Find its infimum and supremum if they exist.

(4mks)

(d) Show that if s is irrational, then $t=\frac{s}{s+1}$ is irrational. (3mks)

(e) Show that an arbitrary union of closed sets need not be closed using the family

$\left\{A_n=\left[\frac{1}{n},1\right]: n\in\mathbb{N}\right\}$ (3mks)

(f) Prove that the sequence $\{x_n\}$ of real numbers has a unique limit if it exists. (4mks)

(g) Investigate the convergence or divergence of the series $\sum_{n=1}^{\infty}\left(\frac{-4}{5}\right)^n$. (3mks)

(h) Show that the function $f(x)=3x-5$ is continuous at $x=5$.. (4mks)

(i) For real numbers x and y show that $\sqrt{xy}\le\frac{1}{2}(x+y)$ where $x.y>0$. (3mks)

QUESTION TWO (20 MARKS)

(a) State the completeness axiom of real numbers. (2mks)

(b) Use mathematical induction to prove that n^3+2n is divisible by 3. (4mks)

(c) Prove that $|x-y|>||x|-|y||$ for every $x,y\in\Re$. (4mks)

(d) Let p be a prime number. Prove that $\sqrt{p}$ is irrational. (5mks)

(e) Let $\{A_i\}_{i=1}^{n}$ be a finite family of open sets. Show that $\bigcap_{i=1}^{n}A_i$ is open. (5mks)

QUESTION THREE (20 MARKS)

(a) Define the following

(i) A convergent sequence. (2mks)

(ii) Determine whether or not the series $\sum\frac{2^n}{n!}$ converges? (3mks)

(b) (i) When is a sequence said to be Cauchy? (2mks)

(ii) Show that every convergent sequence is a Cauchy sequence. (4mks)

(c) Prove that the set of odd integers is countable. (5mks)

(d) Use the definition of the limit of a sequence to prove that $\lim_{n\to 4}2x-5=3$. (4mks)

QUESTION FOUR (20 MARKS)

(a) Define absolute and conditional convergence. (4mks)

(b) Use the integral test to determine the convergence or divergence of $\sum_{n=1}^{\infty}\frac{n^2}{n^3+2}$. (4mks)

(c) Use a suitable test of convergence to evaluate the convergence or divergence of

(i) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{2n^2+1}$. (3mks)

(ii) $\sum_{n=1}^{\infty}\left(\frac{n-1}{n^2}\right)^n$. (3mks)

(iii) $\sum_{n=1}^{\infty}\frac{(-3)^n}{n!}$. (3mks)

(iv) $\sum_{n=1}^{\infty}\frac{(n+1)!}{(n+1)^2}$. (3mks)

QUESTION FIVE (20 MARKS)

(a) Show that the product of two rational numbers is also rational. (3mks)

(b) Let $\{A_i\}_{i\in\Omega}$ be an arbitrary collection of open sets. Show that $\cup A_i$ is open. (4mks)

(c) Let $\{x_n\}$ and $\{y_n\}$ be two sequences converging to *x* and *y* respectively as $n\to\infty$. Then show that $x_n+y_n\to x+y$ as $n\to\infty$. (4mks)

(d) Let $S\subseteq T\subseteq\Re$ and $S\neq\phi$. Show that if *T* is bounded above then $\sup S\leq\sup T$. (3mks)

(e) For any $x, y\in\Re^k$ define the function *d* by $d(x,y)=\sum_{i=1}^{k}|x_i-y_i|$. Show that *d* is a metric on $\Re^k$.

(6mks)