TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

AMA 4306: THEORY OF ESTIMATION

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2 HOURS

DATE: September 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of five questions. Attempt Question and any other two Questions.

Do not write on the question paper.

Question ONE(30marks)

a. A study has been made to compare the nicotine contents of two brands of cigarettes. Ten cigarettes of brand A had an average nicotine content of 3.1 milligrams with a standard deviation of 0.5 milligram while eight cigarettes of brand B had an average nicotine content of 2.7mg with a standard deviation of 0.7mg. Assuming that the two sets of data are independent random variables from normal populations with equal variances, construct a 95% confidence interval for the difference between the mean nicotine contents of the two brands of cigarettes. (6 marks).

b. State the invariance property of a maximum likelihood hence obtain the maximum likelihood estimator of σ given that $\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu_0)^2$ (5marks)

c. Suppose that $x_1, x_2, \ldots, x_n$ is a random sample from a gamma distribution with parameter τ and λ. Given that $E(X) = \frac{\tau}{\lambda}$ and $E(X^2) = \frac{\tau(\tau+1)}{\lambda^2}$ determine the moment estimators of τ and λ (6 marks)

d. The following data was obtained from a Poisson distributed population. Obtain the maximum likelihood estimate of λ ; 12,11,8,9,14,11,15,17,20,16,10,12 and 15 (4marks)

e. Suppose we have a sample of n observations $x_1, x_2, \ldots, x_n$ from an exponential population. Using the method of moments find the estimate of the parameter λ. (4marks)

f. If 132 of 200 male voters and 90 of 150 female voters favor a certain candidate running for a political city, find a 99% confidence interval for the difference between the actual proportions of male and female voters who favor the candidate. (5marks)

Question TWO (20marks)

a. State and Prove Cramer-Rao inequality (12 marks)

b. Let X have a binomial distribution with parameter n and p, obtain the lower bound for the variance an unbiased estimator for p. (8 marks)

Question THREE (20marks)

a. Define the term mean squared error (3 marks)

b. If $x_1, x_2, \ldots, x_n$ constitute a random sample from the population given by

$$f(x) = \begin{cases} e^{-(x-\sigma)} & x > \sigma \\ 0 & elsewhere \end{cases}$$

Show that $\bar{x}$ is a biased estimator of σ. Hence modify the biased estimator to make it unbiased. (8marks)

c. If $x_1, x_2, \ldots, x_n$ constitute a random sample from a normal population with mean μ and variance σ^2, show that $\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is unbiased estimator of the parameter σ^2 (5marks)

d. Obtain the maximum likelihood estimator of λ the parameter in an exponential distribution (4marks)

Question FOUR (20marks)

a. define the term sufficient statistics (2 marks)

b. Suppose we have a sample of n observations $x_1, x_2, \ldots, x_n$ from a Bernoulli population. Show that $Y = \sum_{i=1}^{n} X_i$ is a sufficient statistics for p. (5marks)

c. suppose we random sample of size n from the normal distribution has the density .

$f(x) = \frac{1}{\sigma^2\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}$ show that there is existence of a pair of statistics that are jointly sufficient for μ and σ^2 (7marks)

d. define the term single parameter exponential family hence show that if the random variable X follows a Bernoulli distribution then X is a member of the single parameter exponential family. (6 marks)

Question FIVE (20marks)

a. Define the term simple consistency (3marks)

b. Show that the sequence $\bar{x}_n$ is a mean squared error consistent sequence of estimators of μ (4 marks)

c. A random sample of size n from the normal distribution has the density

$$f(x) = \frac{1}{\sigma^2 \sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x_i - \mu)^2}$$ obtain ;

i. the maximum likelihood function of $f(x)$ (3marks)

ii. the maximum likelihood estimator of μ (3marks)

iii. the maximum likelihood estimator of σ^2 (4 marks)

d. define the admissible estimator (3marks)