TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4313: NUMERICAL ANALYSIS I

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME: 2 HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of FIVE questions. Attempt QUESTION ONE and any other TWO questions.

Do not write on the question paper.

QUESTION ONE Compulsory (30 marks)

a) Define the term interpolation (2 marks)

b) From the following table, find $f(84)$ using Newton's interpolation formula

x	60	70	80	90
$f(x)$	226	250	276	304

(5 marks)

c) Use Lagrange's interpolation formula to find x when $y = 7$ from the following table

x	1	3	4
y	4	12	19

(5 marks)

d) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ using Simpson's $1/3$ rule, n = 6 (4 marks)

e) Given $\frac{dy}{dx} = 1 + xy, y(0) = 1$, compute $y(0.1)$, correct to 4 decimal places using Taylor's series method (6 marks)

f) Use Newton's Raphson method and 3 iterations to obtain the smallest positive root of $f(x) = x^2 - 5x + 2 = 0$ correct to 3 decimal places (5 marks)

g) Let E, ∇ and Δ be shift, backward difference and forward difference operators respectively. Prove that $E^{-1} = 1 - \nabla$ (3 marks)

QUESTION TWO (20 marks)

a) Construct the backward difference table from the data $Sin\ 30^0 = 0.5$, $Sin\ 35^0 = 0.5736$, $Sin\ 40^0 = 0.6428$, $Sin\ 45^0 = 0.7071$. Assuming the 3rd difference to be constant, find the value of $Sin\ 25^0$ (8 marks)

b) Prove that $\left(\frac{\Delta^2}{\mathrm{E}}\right)e^x \times \frac{\mathrm{E}x^2}{\Delta^2 e^x} = e^x$ (7 marks)

c) Given that ;

x	1	2	3	4	5
y	2	5	10	17	26

Find the value of $\nabla^2 y_5$ (5marks)

QUESTON THREE (20 marks)

a) Find the first and second derivatives of the following tabulated function at the point $x = 1.5$

x	1.5	2	2.5	3	3.5	4
$f(x)$	3.375	7.0	13.625	24.0	38.875	59.0

(10marks)

b) Use Newton's method to find a polynomial $p(x)$ of lowest possible degree such that $p(n) = 2^n$ for $n = 0,1,2,3,4$ (7 marks)

c) Construct a divided difference table for the following

x	1	2	4	7	12
$f(x)$	22	30	82	106	216

(3marks)

QUESTION FOUR (20 marks)

a) Using Adam's- Bashforth method find $y(1.4)$ given $y' = x^2(1+y), y(1)=1, y(1.1)=1.233, y(1.2)=1.548$ and $y(1.3)=1.979$ (9 marks)

b) Given $y' = x^2 - y, y(0)=1$, find $y(0.1), y(0.2)$ using Runge kutta method of order two (9 marks)

c) Define the terms argument and entry. (2 marks)

QUESTION FIVE (20 marks)

a) Use the method of iteration to find the root of the equation $x^3 - 5x - 11$ starting with $x_0 = 3$

(8 marks)

b) Solve the differential equation $\frac{dy}{dx} = x + y, y(1) = 0$ by Taylor's series expansion to obtain $y(0.1)$

(6 marks)

c) Use Romberg method to compute $\int_0^1 \frac{dx}{1+x}$ correct to 4 decimal places given

$I(h) = 7084, \quad I(h/2) = 0.6970, I(h/4) = 0.6941$ (6 marks)