AAA

TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHEROR OF SCIENCE STATISTICS AND COMPUTER SCIENCE

YEAR III SEMESTER I

AMA 4304: REAL ANALYSIS

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2HOURS

DATE: September 2018

INSTRUCTIONS: Answer question ONE and ANY OTHER TWO questions.

QUESTION ONE (30 MARKS)

a) Define the following:

(i) Closed Set (1 mark)

(ii) Open Set (1 mark)

(iii) Countable set (1 mark)

(iv) Compact set (1 mark)

(v) bounded set (1 mark)

b) Let A and B be subsets of some universal set U. show that $(A \cap B)^c = A^c \cup B^C$. (4 marks)

c) Prove that $\sqrt{p}$ is an irrational number given that p is prime. (5 marks)

d) (i) State the Completeness Axiom of real numbers. (2 marks)

(ii) Use an example to show that the set of rational numbers does not satisfy the Completeness Axiom. (2 marks)

e) Show that if $x \neq 0, then\ x^2 > 0$ and hence show that $1 > 0$. (3 marks)

f) Use the root test to investigate the convergence or divergence of the series

$$\sum_{n=1}^{\infty} 3^n e^{-n}$$

(3 marks)

g) Show that $H = [0, 1)$ is not a compact set. (4 marks)

h) Show that the set of integers is countable. (2 marks)

QUESTION TWO (20 MARKS)

a) Prove that between any two real numbers, there always exists a third irrational number.

(3 marks)

b) Define the terms infimum and supremum of a set. (2 marks)

c) Find the supremum, infimum, maximal and minimal elements of the sets.

(i) $S = \left\{1 + \frac{1}{n} \middle| n \in \mathbb{N}\right\}$ (4 marks)

(ii) $S = \left\{(-1)^n \left(1 + \frac{1}{n}\right) \middle| n \in \mathbb{N}\right\}$ (4 marks)

d) Show that the intersection of any finite collection of open sets is open in the set of real number. (5 marks)

e) Finite sets have no limit points. Prove. (2 marks)

QUESTION THREE (20 MARKS)

a) Prove that if F is closed subset of a compact set K, then F is compact. (5 marks)

b) Show that the union of countable sets is countable. (5 marks)

c) Show that a non-empty bounded closed subset S of real numbers has minimum element.

(5 marks)

d) Every convergent sequence of real numbers is bounded. (5 marks)

QUESTION FOUR (20 MARKS)

a) If a series is absolutely convergent, then it is convergent. (4 marks)

b) Show that a sequence of real numbers converges to a real number if and only if all its subsequences converge to the same real number. (5 marks)

c) Use the Ratio Test in (i) and Comparison Test in (ii) to investigate the convergence or divergence of the following series. (6 marks)

(i) $$\sum_{n=1}^{\infty} e^{-n} n!$$ (3 marks)

(ii) $$\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$ (3 marks)

d) (i) Given the sequence a_k in $\mathbb{R}$, explain terms:

(a) Absolutely convergent series (1 mark)

(b) Conditionally convergent series. (1 mark)

(ii) Test for absolute or conditional convergence of the series (3 marks)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$$

QUESTION FIVE (20 MARKS)

a) Use Cauchy's criterion to prove the convergence of the sequence

$x_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots\dots + \frac{1}{n^2}$. (4 marks)

b) The limit of a convergent series of a_k is zero as $k \to \infty$. Prove. Hence or otherwise show that $\sum_{n=1}^{\infty} \frac{n}{n+1}$ is not convergent. (5 marks)

c) Show that $f(x) = x^2$ is continuous at $x = a$. (4 marks)

d) Show that $f(x) = x^2 + 1$ is Riemann integrable on $[0,1]$ and find $\int_1^2 (x^2 + 1)\, dx$. (7 marks)