

TECHNICAL UNIVERSITY OF MOMBASA
A Centre of Excellence


## DEPARTMENT OF MATHEMATICS AND PHYSICS

SEPTEMBER 2018 SERIES EXAMINATION
UNIT CODE: AMA 4250 UNIT TITLE: ALGEBRA II

## BTME/BTMA

## SPECIAL/SUPPLIMENTARY EXAMINATION

## TIME ALLOWED: 2HOURS

## INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown
QUESTION ONE (30 MARKS COMPULSORY)
a) Find the inverse of the matrix $M$ where $M=\left(\begin{array}{ll}3 & 2 \\ 2 & 5\end{array}\right)$ and hence solve the matrix
equation $M X=C$ in which $X=\binom{x}{y}$ and $C=\binom{10}{3}$.
(5 marks)
b) Convert $\mathrm{C}_{16}$ in hexadecimal to its decimal equivalent.
c) Solve for $x, y$ and $z$ if $x(5 i+j)+y(j+k)+z k=5 i+3 j-k$.
d) Find the equation of the plane passing through $(3,-1,7)$ and perpendicular to the vector $a=(4,2,-5)$.
e) Let $\mathbf{u}=2 \mathrm{i}-3 \mathrm{j}+4 \mathrm{k}, \mathbf{v}=3 \mathrm{i}+\mathrm{j}-2 \mathrm{k}$ and $\mathbf{w}=\mathrm{i}+5 \mathrm{j}+3 \mathrm{k}$.

Find;

$$
\begin{equation*}
2 u-3 v+4 w \tag{3marks}
\end{equation*}
$$

f) Find $\mathbf{u} . \mathbf{v}$ where $u=(2,-5,6)$ and $v=(8,2,-3)$.
(3 marks)
g) If $\mathbf{A}=3 i-2 j+4 k$ and $\mathbf{B}=2 i-3 j-2 k$. Find $\mathbf{A} \times \mathbf{B}$.
h) Find x for which $\left|\begin{array}{cc}x & 3 \\ 2 & (2 x+1)\end{array}\right|=4$
i) $\quad$ Evaluate $\left|\begin{array}{ccc}1 & 4 & -3 \\ -5 & 2 & 6 \\ -1 & -4 & 2\end{array}\right|$

## QUESTION TWO (20 MARKS)

a) Determine the values of a so that $\mathbf{A}=a i-2 j+k$ and $\mathbf{B}=29 i+a j-4 k$ are orthogonal.
b) Let $\mathbf{u}=(1,-3,4)$ and $\mathbf{v}=(3,4,7)$. Find
i. $\quad d(u, v)$ the distance between $\mathbf{u}$ and $\mathbf{v}$.
ii. $\operatorname{Cos} \theta$, where $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$
c) Solve the following systems of equations.

$$
\begin{aligned}
& 2 x_{1}-x_{2}=7 \\
& -x_{1}+2 x_{2}-x_{3}=1 \\
& x_{2}+2 x_{3}=1
\end{aligned}
$$

Using Gausi - Seidel method of iteration and perform the first five iterations. (8 marks)
d) Resolve the velocity vector of 20 mls at an angle of $-30^{\circ}$ to the horizontal and vertical components.

## QUESTION THREE (20 MARKS)

a) Convert the following binary numbers into their hexagonal equivalent.

$$
\text { (i) } 11010110_{2}
$$

(ii) $1100111_{2}$
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b) Perform the binary addition $1001+10110$
c) Convert the hexadecimal numbers into its decimal equivalent.
$1 \mathrm{~A} 4 \mathrm{E}_{16}$
d) Use crammers rule to solve

$$
\begin{array}{r}
2 y-4 z+6 w=20 \\
3 y-6 z+w=22 \\
-2 y+5 z-2 w=-18
\end{array}
$$

e) Express $w=(1,-2,5)$ as a linear combination of vectors.

$$
\begin{aligned}
& V_{1}=(1,1,1) \\
& V_{2}=(1,2,3) \\
& V_{3}=(2,-1,1)
\end{aligned}
$$

## QUESTION FOUR ( 20 MARKS)

a) Find the solution to the following system of equations
a. $83 x+11 y-4 z=95$
b. $7 x+52 y+13 z=104$
c. $3 x+8 y+29 z=71$

Using Jacobi iterative method for the first five iterations.
(13 marks)
b) Find the angle between the following pairs of planes.
$2 x-y+2 z=3,3 x+6 y+2 z=0$.
c) Find the equation of the plane passing through the point $(1,2,-1)$ and perpendicular to the planes $x+y-2 z=5$ and $3 x-y+4 z=12$.

## QUESTION FIVE (20 MARKS)

a) Given $A=\left[\begin{array}{ll}1 & 2 \\ 3 & 5\end{array}\right] \quad B=\left[\begin{array}{ll}2 & 1 \\ 1 & 3\end{array}\right]$ show that $(A B)^{\top}=B^{\top} A^{\top}$
b) Determine the values of x for which the determinant of A is zero where;

$$
\mathrm{A}=\left[\begin{array}{ccc}
x-2 & 4 & 3 \\
1 & x+1 & -2 \\
0 & 0 & x-4
\end{array}\right]
$$

c) Use logic gates to represent these expressions and draw the corresponding truth tables.
j) $\quad \sim p \vee q$
ii) $(x \vee y) \wedge \sim x$

