TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BMCS/BSSC2016

AMA 4217: LINEAR ALGEBRA 1

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: **September2018**

TIME: 2 HOURS

DATE: **September2018**

<u>Instructions to Candidates</u>

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

SECTION A

QUESTION ONE (30 MARKS)

(a) Verify that if $M = \begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix}$ and N= $\begin{pmatrix} -1 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 0 & 5 \end{pmatrix}$, then $MN = NM = I$ where I is a unit matrix. Use the above information to solve the matrix equation

$$\begin{pmatrix} -5 & 10 & 8 \\ 4 & -7 & -6 \\ -3 & 6 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 2 \end{pmatrix}$$ (5 marks)

(b) Solve the linear system

$$\begin{aligned} -p + 2r + 4s &= 7 \\ 2p + r - 2s &= -2 \\ -3p \qquad +5s &= 7 \end{aligned}$$ (6 marks)

(c) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 2 \end{pmatrix}$, determine the classical adjoint of A hence find the inverse of A . (6 marks)

(d) Reduce the matrix $\begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix}$ to row reduced echelon form. (6 marks)

(e) Find A^{T} and $\left(\mathrm{A}^{\mathrm{T}}\right)^{\mathrm{T}}$ given that $\mathrm{A} = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & -2 & 1 & 2 \end{pmatrix}$. (3marks)

(f) Find
$(i)\ (1,2)\bullet(2,3)$
$(ii)\ (2i+j)\times(3i-j)$ (4marks)

SECTION B

QUESTION TWO (20 MARKS)

(a) M and N are members of a set S which is defined as $S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix}, a, b \in \Re \right\}$. Show that the product of M and N is also a member of S

(5 marks)

(b) Determine the distance between the planes $x+2y-2z=3$ and 2x+4y-4z=7 (5 marks)

(c) Find the dimension and a basis for

(i) $U=\{(x,y,z,w,t)/x+y+z+w+\text{t}=0 \text{ and x-y+z-w+t=0}\}$ (5 marks)

(ii) $V=\{(x,y,z)/x=2y \text{ and z=3y}\}$ (5 marks)

QUESTION THREE (20MARKS)

(a) Find the angle between the planes $2x+y-2z=1$ and x-2y-2z=2 (6marks)

(b) Determine the equation of the plane through the point $(1,2,3)$ and perpendicular to the planes $2x-3y+4z=1$ and 3x-5y+2z=3 (7 marks)

(c) Solve the given system by the Gauss Elimination method.

$$\begin{aligned} x+y+z+w&=4 \\ x+2y-z-w&=7 \\ 2x-y-z-w&=8 \\ x-y+2z-2w&=-7 \end{aligned}$$

(7 marks)

QUESTION FOUR (20 MARKS)

(a) If $\begin{vmatrix} \lambda & \lambda \\ 3 & \lambda-2 \end{vmatrix}=0$, find λ. (3marks)

(b) Find the distance from the origin to the plane $2x+3y-z=2$. (3maks)

(c) Write the line $\frac{x-3}{2}=\frac{y+4}{3}=\frac{5-z}{4}$ in the form **v**=a+ t**u** and show it passes through (1,-7,9) (7 marks)

(d) Write the polynomial $v=t^2+4t-3$ as a linear combination of the polynomials $e_1=t^2-2t+5,\ \text{e}_2=2t^2-3t$ and $\text{e}_3=t+3$ (7 marks)

QUESTION FIVE (20 MARKS)

(a) Suppose that $\vec{v}_1, \vec{v}_2, ...\vec{v}_k$ are non zero vectors such that $\vec{v}_i . \vec{v}_j = 0$ whenever $i \neq j$.

Show that $\vec{v}_1, \vec{v}_2, ...\vec{v}_k$ are linearly independent. (5 marks)

(b) Show that $\{(1,1,1),(0,1,1),(0,1,-1)\}$ span $\mathbb{R}^3$. (5marks)

(c) Determine whether or not $\{(1,1,1),(1,2,3),(2,-1,1)\}$ form a basis for $\mathbb{R}^3$. (5marks)

(d) Find the dimension and a basis for $W = \{(a,b,c,d) / b - 2c + d = 0\}$ (5 marks)