TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

FOR THE FIRST SEMESTER IN THE SECOND YEAR OF BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE, BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS & FINANCE

AMA 4210: PROBABILITY &STATISTICS II

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME:2HOURS

DATE: Pick DateSep2018

<u>Instructions to Candidates</u>

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attemptquestion ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

SECTION A (COMPULSORY)

Question1 (30 marks)

(a) Define the following terms:
  - (i) Random variable
  - (ii) parameter
  - (iii) Random experiment

(iv) Sample space (4 marks)

(b) Let X be a discrete random variable with distribution

X	0	1	2
P(X=x)	3/8	¼	3/8

Find:

(i) P(X=0 or X=1) (2 marks)

(ii) Mean and variance of X (4marks)

(c) A lot of size 100 contains 50 defective articles. Suppose that a sample of 10 articles is drawn at random from the lot, find:

(i) The probability mass function of the number of defectives, X(2 marks)

(ii) The probability that the sample contains less than 2 defectives(4 marks)

(d) The mean weight of 500 packets of sugar is found to be 1012g. of the 500 packets, 35 were found to have a weight in excess of 1015 g. Assuming the weights are normally distributed about the mean, estimate :

(i) The standard deviation of the weights (3 marks)

(ii) The number of packets weighing less than 1008g (2 marks)

(e) Let X be a continuous random variable with probability distribution

$$f(x) = \begin{cases} \frac{x}{12}, & 1 < x < 5 \\ 0, & elsewhere \end{cases}$$

Find the probability distribution of the random variable $Y = 2X - 3$ (6marks)

(f) Find the mean, variance and standard deviation of a binomial random variable with n=10, p=0.8 (3 marks)

SECTION B (Answer any TWO questions from this section)

Question2 (20 marks)

(a) Find the moment generating function of a random variable whose probability density function is given by

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & elsewhere \end{cases}$$ (6 marks)

(b) Using the moment generating function of the random variable in (a), find:

(i) the mean μ (2 marks)

(ii) the second moment μ_2' (2 marks)

(iii) the variance μ_2 (2 marks)

(c) The number of cars, X, that pass through a car wash between 4.00p.m. and 5.00 p.m. on any sunny Friday has the following probability distribution:

X	4	5	6	7	8	9
P(X=x)	1/12	1/12	¼	¼	1/6	1/6

Let g(X) =2X-1 represent the amount paid in K£ paid to the attendant by the manager. Find:

(i) The attendant's expected earnings for this particular time (3 marks)
(ii) The variance of the attendant's earnings for the given period(5 marks)

Question3 (20 marks)

(a) Compilation of a computer program consists of 3 blocks that are processed sequentially, one after the other. Each block takes an exponential time with mean of 5 minutes, independently of other blocks. Compute:
   (i) The expectation and variance of the total compilation time (4 marks)
   (ii) The probability for the entire program to be compiled in less than 12 minutes (7 marks)

(b) The Kenya Revenue Authority is a body mandated to collect tax on behalf of the Kenya government. If the annual proportion of erroneous tax returns filed with KRA is found to be a random variable having a beta distribution with $\alpha = 6$ and $\beta = 9$, determine:
   (i) The mean of erroneous tax returns (3 marks)
   (ii) The probability that there will be less than 10% erroneous tax returns (6 marks)

Question4 (20 marks)

(a) A shipment of 7 computers contains 2 computers suspected to be defective. The IT workshop makes a random purchase of 3 computers. If X is the number of defective computers bought by the workshop;
   (i) Express the results graphically on probability histogram (7marks)
   (ii) Find the CDF of X (2marks)

(b) Using (a)(iii), determine:
   (i) P(X=1) (1 mark)
   (ii) P(0<x<2) (1 mark)

(c) The time to failure of a certain brand of electric bulb can be represented by the density function

$$f(x) = \begin{cases} \frac{1}{2000} e^{-\frac{1}{2000}x}, & x > 0 \\ 0, & elsewhere \end{cases}$$

Determine:

(i)	F(x)	(2 marks)
(ii)	The probability that the bulb lasts more than 100hours	(1 mark)
(iii)	The probability that the bulb fails before 2000 hours	(1 mark)
(iv)	The time to failure of the bulb	(5 marks)

Question5 (20 marks)

The table below shows the joint distribution of two random variables, X and Y.

		Values of Y			
		1	2	3	4
	1	6c	3c	2c	4c
	2	4c	2c	4c	0
Values of X	3	2c	*c*	0	2c

(a) Find c. (2marks)

(b) Calculate the marginal distributions of X and Y. (4marks)

(c) Calculate $E(X)$ and $\text{Var}(X)$, and show that the covariance $\text{Cov}(X, Y) = 0$.

(7marks)

(d) State, with a reason, whether or not X and Y are independent.(2marks)

(e) The random variables U and V are defined by

$U = 1$ if $X = 1$ or 3, $U = 0$ if $X = 2$,

$V = 1$ if $Y = 1$ or 3, $V = 0$ if $Y = 2$ or 4.

Tabulate the joint distribution of U and V and state with a reason whether or not U and V are independent (5 marks)