TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

THE SECOND YEAR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND FINANCE, CIVIL, MECHANICAL, ELECTRICAL AND ELECTRONICS, MEDICAL ENGINEERING.

AMA 4209: CALCULUS III

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER- 2018

TIME: 2 HOURS

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 5 questions. Attempt Question One and any other Two Questions.

Use SMP four figure mathematical tables and non-programmable electronic calculators.

Do not write on the question paper.

Question ONE (30 marks) compulsory.

a) State all conditions that a function f(x) must satisfy for it to be continuous at x= a. (2 marks)

b) Show that the sequence $a_n = \frac{n^2}{2^n - 1}$ converges and determine the limit. (6 marks)

c) Solve the improper integral $\int_1^\infty (1-x)e^{-x}dx$ (5marks)

d) Two stationary patrol cars with radars are 5km apart on a high way and a truck passes the first patrol car, its speed is clocked at 55km/h. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50km/h. Prove that the truck must have exceeded the speed limit of 60 km/h at some point during the interval. (3 marks)

e) Using the fourth degree Maclaurin polynomial, approximate $\ln(1.1)$. (6 marks)

f) Calculate $\iint_R x\cos(xy)dA$, where R is the rectangle $\{(x,y) \mid 0 \le x \le \frac{\pi^c}{4},\ 0 \le y \le 2\}$ (4marks)

g) Show that any differential function of the form $w = f(s)$ where $s = y + bx$ is a solution of the partial differential equation $\frac{\partial w}{\partial x} - b\frac{\partial w}{\partial y} = 0$ where $b \ne 0$ is a constant. (4 marks)

Question TWO (2O MARKS)

a) Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (1,2) for $f(x,y) = \sqrt{9 - x^2 - y^2}$ (5 marks)

b) Find the amount of money you should invest today at an annual interest rate of 10% compound continuously if $A = Pe^{-rt}$ so that starting next year, you can make annual withdrawals of Kshs 400 in perpetuity. (7 marks)

c) Let $\phi = 45x^2y$ and V denotes the closed region bounded by the planes $4x + 2y + z = 8$, $x = 0$, $y = 0$, $z = 0$. Evaluate the volume integral $\iiint_V \phi dV$ of the solid. (8 marks)

Question THREE (20 MARKS)

a) If $f(x) = x^3 + 1$ show that f(x) satisfies the mean value theorem on the interval [1, 2] and find the value of C whose existence is guaranteed by the theorem. (4 marks)

b) Find the moments and centre of mass for a system of objects that have masses 3, 4 and 8 at the points (-1,1), (2,-1) and (3,2). (5 marks)

c) Evaluate $\iint_T 6xydA$ where T is the region in the first quadrant bounded by two graphs $y = x^2$ and $y = 2x$. (5 marks)

d) Obtain the Taylors polynomial of degree 6 for $f(x) = \cos x$ about $x = 0$. Use your answer to approximate $\cos 1$. (6 marks)

Question FOUR (20 Marks)

a) Find the radius of convergence of a power series $\sum_{n=0}^{\infty} 3(x-2)^n$. (4 marks)

b) Find a sequence $\{a_n\}$ whose first five terms are $\frac{2}{1}, \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \ldots$. and determine whether it converges or diverges.

. (6marks)

c) Let R be the square $\{(x,y) \mid -1 \le x \le 1, \quad -1 \le y \le 1\}$. Calculate the volume of the solid region determined by the graph of $f(x,y) = 8 - x^2 - y^2$ over R. (6 marks)

d) Determine $\left.\frac{df}{dt}\right|_{t=0}$ if f (x,y,z) = xy+z where x=cost , y= sin t and z = t. (4 marks)

Question FIVE (20 marks)

a) Use geometric series to express $0.08\dot{0}\dot{8}$ as a ratio of two integers. (5 marks)

b) Find the Taylor polynomials P_0, P_1, P_2, P_3 and P_4 for $f(x) = \ln x$ contained at $c = 1$.

(5 marks)

c) Using L'Hopital's rule, evaluate $\lim_{x \to 0}\left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$ (5 marks)

d) Find the area of the region enclosed by the graph of a polar equation $r = 5\sin 3\theta$ which is a 3-leaved flower.

. (5 marks)

THE **END**