TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BSSC/BMCS/BSMF YEAR II SEMESTER I

AMA 4208: ALGEBRAIC STRUCTURES

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September 2018

TIME: 2HOURS

DATE: September 2018

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Let $\mathcal{G} = \{a, b, c\}$ and let the binary operation $*$ on G given on the table below

$*$	a	b	c
a	a	b	c
B	b	c	a
C	c	a	b

i) Find the identity in G (1 Mark)

ii) Find the inverse of c (1 Mark)

iii) Is G commutative? (1 Mark)

b) Define the order of a group (2 Marks)

c) Prove that every field is an integral domain. (4 Marks)

d) Define a subring. (3 Marks)

e) Find all the distinct left coset of $5\mathbb{Z}$ in $\mathbb{Z}$ (5 Marks)

f) Outline the group of symmetries of an equilateral triangle (3 Marks)

g)

i) Define a permutation (2 Marks)

ii) Given that $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ $\delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}$

Find $\delta\tau$ as a product of disjoint cycles (4 Marks)

h) Let $G = \mathbb{Z}_5 \backslash \{0\}$ be a group under multiplication Modulo 5. Draw the Cayley table for G. (4 Marks)

QUESTION TWO (20 MARKS)

a) Let $\mathbb{N} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ be a subset of a group of 2×2 invertible matrices $\mathcal{M}_2(\mathbb{R})$

$\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} / ad - bc \neq 0\right\}$. Show that $\mathbb{N}$ is a subgroup of $\mathcal{M}_2(\mathbb{R})$ (5 Marks)

b)

i) Define an identity element (1 Mark)

ii) Show that the identity element in a set Sunder the binary operation * is unique.

(4 Marks)

c) Let H and K be subgroup of G. Show that $H \cap K$ is also a subgroup of G. (6 Marks)

d) Let G be the Quartenion group; find the cyclic subgroup of G generated by;

i j (2 Marks)

ii k (2 Marks)

QUESTION THREE (20 MARKS)

a) Define a ring (5 Marks)

b) Prove that if R is a ring with unity, then R has a characteristic $n > 0$ if and only if n is the least positive integer such that $n.1 = 0$ (5 Marks)

c) Show whether the following are integral domains

(i) The set of integers (1 Mark)

(ii) $\mathbb{Z}_8$ (1 Mark)

(iii) $3\mathbb{Z}$ (1 Mark)

(iv) $\mathbb{Z}_7$ (1 Mark)

j) Prove that multiplicative cancellation laws hold in a ring R if and only if R has no zero divisors. (6 Marks)

QUESTION FOUR (20 MARKS)

a) State whether the following statements are true or false.

i) $\mathbb{Z}, \mathbb{Q}, \mathbb{N}, \mathbb{R}$ *and* $\mathbb{C}$ are associative under addition and multiplication (1 Mark)

ii) If P is prime, then $\mathbb{Z}_p$ is a field (1 Mark)

iii) The set $\mathbb{Z}_6$ has no nilpotent elements (1 Mark)

iv) All zero divisors are integral domains (1 Mark)

v) A multiplicative identity in a ring is called the order of the ring (1 Mark)

b) Prove that the order of an element of a finite group divides the order of a group (5 Marks)

c) Define the following terms

i) A nilpotent element in a ring R (2 Marks)

ii) A unit in a ring (2 Marks)

d) Prove that every finite integral domain is afield. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Let R be a ring, prove that for any $a, b \in R$ we have;

i) $0a = a0 = 0$ (2 Marks)

ii) $(-a)(-b) = ab$ (3 Marks)

b) Define:

(i) A composition of functions (2 Marks)

(ii) Cyclic subgroup (2 Marks)

c) Prove that in the ring $\mathbb{Z}_n$, the zero divisors are precisely those elements that are not relatively prime to n. (4 Marks)

d) State and prove Lagrange's theorem (7 Marks)