TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BSMD/BTMD/BTRE/BTAP2016

AMA 4206: LINEAR ALGEBRA

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: September2018

TIME: 2 HOURS

DATE: September2018

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE 30 MARKS [COMPULSORY]

a) Verify that the matrix

i. $A = \begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix}$ Is ***idempotent***

[1 mark]

ii. $B = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ Is ***nilpotent*** of order 3

[2 marks]

b) i. State ***Sylvester's law*** of Nullity

[2 marks]

iii. Given $T(x) = \begin{pmatrix} 4 & 1 & -2 & -3 \\ 2 & 1 & 1 & -4 \\ 2 & 0 & -9 & 9 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, find the **nullity**, **rank** and **basis** of ker(T)

[7 marks]

c) Find x, y and z in the system using **Cramer's** rule $\begin{matrix} -2x + 3y - z = 1 \\ x + 2y - z = 4 \\ -2x - y + z = -3 \end{matrix}$

[4 marks]

d) i. Given the compacted system of Cartesian equations ***Ax***=***b***. State the conditions for each of the following possibilities to happen to the solution

I . Unique solution

[1 mark]

II. No solution

[1 mark]

III. Infinitely many solutions

[1 mark]

ii. Determine the value of '***a***' so that the system has $\begin{matrix} x - 3y = -3 \\ 2x + ay - z = -2 \\ x + 2y + az = 1 \end{matrix}$

I. **No** solution

[5 marks]

II. **Infinitely** many solutions

[1 mark]

III. **Unique** solution

[1 mark]

e) Find the **cross product** of the vectors $\tilde{a} = 3i - 5j + k \; and \; \tilde{b} = 2j - 4k$

[4 marks]

QUESTION TWO (20 MARKS)

a) Suppose that ***V*** is a vector space and $S = \{v_1, v_2, v_3 \ldots \ldots . .\}$ is a set of vectors in **V**

i. Define a basis for **V**

[2 marks]

ii. Determine if $S = \{(1,2,1), (2,9,0), (3,3,4)\}$ is a basis for $\mathbb{R}^3$

[8 marks]

b) Find the **acute** angle between the diagonals of a quadrilateral having vertices at (0,0,0), (3,2,0), (4,6,0) and (1,3,0)

[6 marks]

c) Find the ***determinant*** of Matrix A given by $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$ using the Laplace method

[4 marks]

QUESTION THREE (20 MARKS)

a) Find $W\left(sinx;\; sin2x; \frac{\pi}{4}\right)$

[4 marks]

b) Obtain the reduced **echelon** form of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

[3 marks]

c) For any vectors ***U,V***$\in \mathcal{R}^n$ and any scalars a,$b \in \mathcal{R}$, with S as a field. Prove that

i. $(a+b)u = au + bu$

[2 marks]

ii. $(ab)u = a(bu)$

[2 marks]

iii. $u + (-u) = 0$

[2 marks]

d) If $A = xz^3 i - 2x^2 yzj + 2yz^4 k$. Determine $\nabla X A$ at (1,-1,1)

[5 marks]

e) Define the terms

i. Elementary matrix

[1 mark]

ii. Homogeneous system of equations

[1 mark]

QUESTION FOUR (20 MARKS)

a) Solve the ***determinantal*** equation $det \begin{vmatrix} 1 & 1+x \\ 1+x & 1 \end{vmatrix} = 0$

[3 marks]

b)

i. Let $k_1 v_1 + k_2 v_2 + k_3 v_3 + \cdots \ldots \ldots + k_n v_n = 0$ be a ***homogeneous*** system of equatons with at least one solution; then differentiate linearly **dependence** and **independence** of the set of vectors = $\{v_{1,} v_{2,} \ldots v_n\}$

[2 marks]

ii. Determine if $S = \{(2 + x + x^2), (x - 2x^2), (2 + 3x - x^2)\}$ is ***linearly independent*** in P_2

[6 marks]

c) Find A^{-1} (the inverse of the matrix A) by first getting the **ad joint** of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 1 & 0 & 3 \end{pmatrix}$

[6 marks]

d) If $\emptyset(x, y, z) = 3x^2y - y^3z^2$, Find $\nabla\emptyset$ at (1,-2,-1)

[3 marks]

QUESTION FIVE (20 MARKS)

a) Determine if $\boldsymbol{T}: \mathbb{R}^{\mathbf{2}} \to \mathbb{R}^{\mathbf{2}}$ defined by $T(x_1, x_2) = (x_1 + 1, x_2)$ is a ***linear transformation***

[4 marks]

b) Partition the matrix into four equal sized 2 x 2 matrices and compute the ***determinant***

$$\begin{pmatrix} 1 & 4 & 0 & 2 \\ 3 & 2 & 1 & -2 \\ 4 & 4 & 3 & 5 \\ -5 & 3 & 2 & 0 \end{pmatrix}$$

[6 marks]

c)

i. Define a ***subspace*** W if V is a vector space and W be a **non-empty** subset of V

[2 marks]

ii. Find the ***basis*** and ***dimension*** Space for the equations

$$\begin{gathered} 2x_1 + 2x_2 - x_3 + x_5 = 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0 \\ x_1 + x_2 - 2x_3 - x_5 = 0 \\ x_3 + x_4 + x_5 = 0 \end{gathered}$$

[8 marks]