

# TECHNICAL UNIVERSITY OF MOMBASA 

A Centre of Excellence
 DEPARTMENT OF MATHS AND PHYSICS

## SERIES: September2018

AMA 4203: STATISTICS
TIME ALLOWED: 2HOURS
INSTRUCTIONTO CANDIDATES:
You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown

## QUESTION ONE (30 MARKS COMPULSORY)

a. Differentiate between each of the following
i. Descriptive and inferential statistics
(2Marks)
ii. Continuous and discrete data (2Marks)
b. In one game the chance that a certain soccer team will win, draw or lose is $0.8,0.02$ and 0.18 respectively. The team plays two games
i. Represent this on a tree diagram
ii. Hence determine the probability of winning both games
(2Marks)
c. A random sample of 10 items is taken and found to have a mean weight of 60 grams and a standard deviation of 12 grams. What is the proportion of the population?
i. With $95 \%$ confidence?
(2Marks)
ii. With 99 \% confidence?
d. State two characteristics of the Poisson distribution
e. The following data has been collected regarding sales and advertising expenditure

| Sales ( $£$ ' M ) | Advertising expenditure ( $£^{\prime} 000$ ) |
| :--- | :--- |
| 8.5 | 210 |
| 9.2 | 250 |
| 7.9 | 290 |
| 8.6 | 330 |
| 9.4 | 370 |
| 10.1 | 410 |

i. Plot the data on a scatter diagram
(2Marks)
ii. Decide whether there is a correlation between sales and advertising expenditure 2Marks
f. The PDF of a random variable is given as

$$
f(x)=\left\{\begin{array}{cc}
\frac{2 x+1}{6}, & 0 \leq x \leq 2 \\
0, & \text { elsewhere }
\end{array}\right.
$$

Determine
i. $E(X)$
(2Marks)
ii. $\quad E\left(X^{2}\right)$
(3Marks )
g. The information in the table below relates to the performance of students

| Class | $0-20$ | $20-40$ | $50-60$ | $60-70$ | $70-80$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 0 | 5 | 17 | 28 | 12 |

Calculate the geometric mean using the above data
(4 Marks)
h. Analysis of questionnaires completed by holiday makers showed that 0.75 classified their holiday as good at Costa Lotta. The probability of hot weather in the resort is 0.6 . If the probability of regarding the holiday as good given hot weather is 0.9 . What is the probability that there was hot weather if a holiday maker considers his holiday as good?
(4Marks)

## QUESTION TWO (20 MARKS)

a. What is the value of
i. The first Quartile
ii. The third Quartile
iii. The interquartile range
iv. The telephonist answers telephone calls arriving at the switchboard of a particular
v. Organisation. A random sample of 100 calls received at the switchboard on a particular day was monitored and the time taken for the telephonist to answer was recorded. The data obtained are summarized in the following table.

## Time in seconds

| $\geq 10$ but $<20$ | 16 |
| :--- | ---: |
| $\geq 20$ but $<25$ | 10 |
| $\geq 25$ but $<30$ | 20 |
| $\geq 30$ but $<35$ | 21 |
| $\geq 35$ but $<40$ | 14 |
| $\geq 40$ but $<50$ | 10 |
| $\geq 50$ but $<70$ | 4 |
| $\quad$ Total | 100 |

$\geq 20$ but $<25$
$\geq 25$ but < 30
Number of Calls
$\geq 30$ but < 35
21
$\geq 35$ but < 40 10
$\geq 40$ but < 50

100
vi. (a) Draw a histogram depicting the above data.
(4marks)
vii. (b) Estimate the mean (4marks)
viii. (c) Estimate the median of the data. (3marks)
ix. (d) What do the data and your statistics indicate about the distribution of the number of seconds it takes for the telephonist to answer a call? ( 2 marks)
x. (e) Construct a $95 \%$ confidence interval for the mean number of seconds for a call to be answered
(4marks)

## QUESTION THREE (20 MARKS)

a. A random variable $X$ has the following probability function

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X}=\mathrm{x})$ | 0 | k | 2 k | 2 k | 3 k | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

i. Find the value of $k$
(2 Marks)

Evaluate
ii. $\quad P(X<6)$
iii. $\quad P(X \geq 6)$
iv. $\quad P(0<X<5)$
v. If $P(X \leq a)>\frac{1}{2}$

Find the minimum value of a
(2 Marks)
b. Determine the distribution function of $X$
(3 Marks)
c. Two dice are rolled. Let $X$ denote the random variable which counts to the total number of points on the upturned faces.
i. Construct a table giving non -zero values of the PMF (2Marks)
ii. Draw the probability chart
(2Marks)
iii. Find the distribution function of $X$
(3Marks)

## QUESTION FOUR (20 MARKS)

Torch bulbs are packed in boxes of 5 and 100 boxes are selected to test for the number of defectiveness

| Number of defectives | Number of boxes | Total defectives |
| :--- | :--- | :--- |


| 0 | 40 | 0 |
| :--- | :--- | :--- |
| 1 | 37 | 37 |
| 2 | 17 | 34 |
| 3 | 5 | 15 |
| 4 | 1 | 4 |
| 5 | 0 | 0 |

The probability of any individual bulb being rejected is $\frac{90}{100} \times 5=0.18$ and it is required to test at the $5 \%$ level whether the frequency of the rejects conforms to the binomial distribution.
a. State the hypothesis (2Marks)
b. State the binomial expansion in terms of $p$ and $q$
c. Summarize your calculations to complete the table below
(14 Marks)

| Defectives | No. of boxes | Binomial | Expected |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| probabilities | Frequency | $(O-E)^{2}$ | $\frac{(O-E)^{2}}{E}$ |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

## QUESTION FIVE (20 MARKS)

Batches of 5,000 electric lamps have a mean life of 1,000 hours and a standard deviation of 75 hours. Assume a normal distribution
a. How many lamps will fail before 900 hours? Explain
(3 Marks )
b. How many lamps will fail between 950 and 1000 hours?
(3 Marks)
c. What proportion of the lamps will fail before 925 hours?
(3 Marks )
d. Given the same mean life, what would the standard deviation have to be in order to ensure that not more than $20 \%$ of the lamps fail before 915 hours? (3 Marks)

A firm buys springs in very large quantities from past records; it is known that $0.2 \%$ are defective. The inspection department sample the springs in batches of 500. It is required to set a standard for the inspectors so that if more than the standard number of defectives is found in a batch, the consignment can be rejected with at least 90\% confidence that supply is truly defective. How many defectives per batch should be set as the standard?

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