TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

SEPTEMBER 2018 SERIES EXAMINATION

AMA 4160: PURE MATHEMATICS

BTEE

TIME ALLOWED: 2 HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as show

QUESTION ONE (30 MARKS) COMPULSORY

(a) Solve the equation $2.68 = \ln\left(\frac{4.87}{x}\right)$ (3 marks)

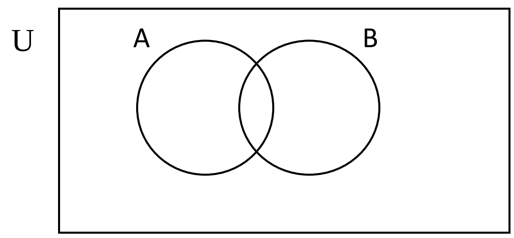
(b) The universal set U is defined as a set of positive integers less than 10. The subsets A and B are defined as

A = {integers that are multiples of 3}

B = {integers that are factors of 30}

a) List the elements of
   (i) A (1 mark)
   (ii) B (1 mark)

b) Place the elements of A and B in the appropriate region in the venn diagram below



(2 marks)

c) If (x – 2) , (x + 2) and (5x – 2) are consecutive terms of a geometric sequences. Find the common ratio and the first term. (4marks)

d) solve by completing the square method
   $x^2 + 4x - 12 = 0$ (3marks)

e) solve for θ in the range $0^o \leq \theta \leq 360^o$ for $2 \sin\theta = \cos\theta$ (4marks)

f) let f_1 and f_2 be functions from the set of real numbers to the set of real numbers such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. find :
   (i) $f_1 + f_2$
   (ii) $f_1 f_2$ (4 marks)

g) let **u** = (2,4,-5) and **v** = (1,-6,9) find:
   (i) 3**u** – 5**v**
   (ii) $\|3u - 5v\|$ (5 marks)

h) given that log 2 =0.3010 and $\log 3 = 0.4771$ *find* $\log 12$ (3marks)

QUESTION TWO (20 MARKS)

(a) (i) Given that $S_n = a + ar + ar^2 + \ldots\ldots ar^{n-1}$ show that the sum of the geometric progression is given by

$$S_n = a\left(\frac{r^{n} - 1}{r-1}\right)$$ (4marks)

(ii) A drilling machines is to have 6 speeds ranging from 50 rev/min to 750rev/min. If the speed forms a geometric progression determine their values correct to two decimal places. (6marks)

(b) Given that f(x) = 5x + 1 and g(x) = x^2. Express the composite fog and gof in their simplest form possible and find f(g(-2)) and g(f(3)). (6marks)

(c) Show that the functions f(x) = 2x + 3 and g(x) = ½(x -3) are inverses of each other. (4marks)

QUESTION THREE (20 MARKS)

(a) The root of a quadratic equation $4x^2 + 8x - 1 = 0$ are α and β , determine the values of α and β (3 marks)

(b) Simplify $(5^{-1})^2 \times 5^4 \times (5^2)^{-2}$ (3 marks)

(c) Find the remainder when $x^5 + 4x^3 + 2x + 3$ is divided by x + 2

i) By direct polynomial division (3marks)

ii) Using the remainder theorem (2marks)

d) Find the solution of the equation for $0 \leq \theta \leq 360$

$$2\cos^2\theta - \sin\theta = 1$$ (4marks)

di) Solve the simultaneous equation

$$\log x - 3\log y = 1$$ (5marks)

$$xy = 160$$

QUESTION FOUR (20 MARKS)

(a) let **u**= $\begin{pmatrix} 5 \\ 3 \\ -4 \end{pmatrix}$; **v** = $\begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix}$, **w** = $\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$

find (i) 5**u** – 2**v**

(ii) -2**u** + 4**v** – 3**w** (4 marks)

(b) let u = (2, -5, 6) and v = (8, 2, -3)

find **u.v** (3 marks)

(c) Find k so that **u** and **v** are orthogonal where **u** = (1,k,-3) and **v** = (2, -5,4) (3 marks)

(d) A market researcher investigating consumers preference for three brands of beverages namely; coffee, tea and cocoa, in Kisii town gathered the following information. from a sample of 800 consumers, 230 took coffee, 245 took tea and 325 took cocoa, 30 took all the three beverages, 70 took coffee and cocoa, 110 took coffee only, 185 took cocoa only.

Required :

(i) Present the above information in a venn diagram

(ii) Find the number of customers who took tea only

(iii)Find the number of customers who took coffee and tea only.

(iv)Find the number of customers who took tea and cocoa only.

(v) The number of customers who took none of the beverages. (10 marks)

QUESTION FIVE (20 MARKS)

a) Find in terms of x the number of terms and the sum of the arithmetic series

$(2x + 1) + (2x + 3) + (2x + 5) + \ldots\ldots\ldots + (4x + 9)$ (6 marks)

b) Convert the following into the sum of partial fractions

i) $\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$ (4 marks)

ii) $\frac{11-3x}{x^2+2x-3}$ (3marks)

c) Solve for x

i) $(x-1)^{\frac{3}{4}} = 64$ (2 marks)

ii) $4^x - 2^x - 12 = 0$ (5marks)