TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

DECEMBER 2016 SERIES EXAMINATION

UNIT CODE: AMA 4102 UNIT TITLE: APPLIED MATHEMATICS 1

EXAMINATION FOR BACHELOR OF TECHNOLOGY IN ELECTRICAL AND ELECTRONICS ENGINEERING

MAIN EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

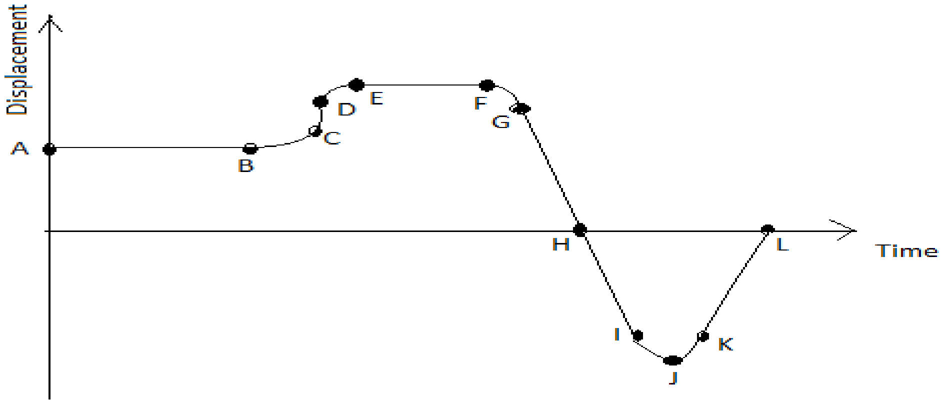
Mobile phones are prohibited in the examination hall

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. Describe the motion represented by the figure of Displacement against time below



[4 marks]

b. Given that $\vec{F} = (2xy + z^3)i + x^2j + 3xz^2k$

i) Show that F is a **conservative** force field [2 marks]

ii) Find the scalar potential [2 marks]

iii) Find the **work done** in moving an object in this field from (1,-2,1) to (3,1,4) [2 marks]

c. If. $A = x^2zi - 2y^2z^2j + xy^2zk$

Find $\nabla . A$ at the point (1, -1, 1) [4 marks]

d. A nozzle is situated at a distance of 1.2 m above the ground level and is inclined at 60^0 to the horizontal. The diameter of the nozzle is 40 mm and the jet of water from the nozzle strikes the horizontal distance of 5m. Find the initial velocity u. [6 marks]

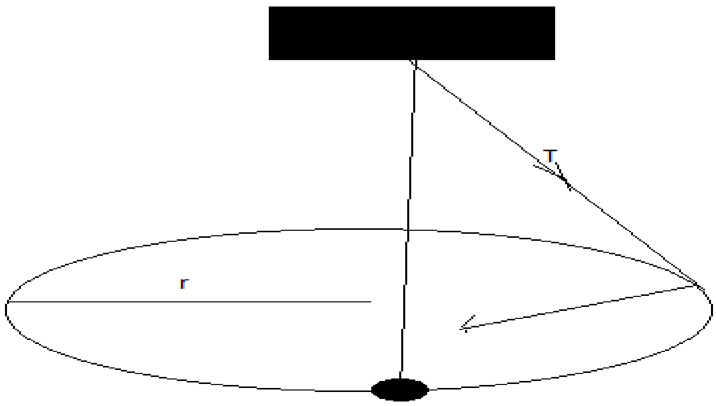
e. Find the power exerted by a force $F = 5t^2i + 2tj$ on a particle with velocity $v = ti - 2t^2j$ when t=16 seconds. [3 marks]

f. A slender metal arch, thicker at the bottom than at the top, lies along the semi circle $y^2 + z^2 = 1,\ z \geq 0,$ in the y-z plane. Find the centre of the arch's mass if the density at the point (x, y, z) on the arch is $\delta(x, y, z) = 2 - z$ [7 marks]

QUESTION TWO (20 MARKS)

a. For a conical pendulum as shown below. Prove that for unit radius r of the circular path $\tan\theta = \frac{v^2}{g}$ where is θ the angle of inclination

[4 marks]



b. Differentiate coplanar forces from concurrent forces. [2 marks]

c. Given the space curve $x = t, y = t^2,\ z = \frac{2}{3}t^3$ find

   i) The curvature k and radius of curvature ρ [5 marks]

   ii) The tortion τ [4 marks]

d. A particle is moving along a straight line according to the law $= 4t^3 + 3t + 2$. if the distance is x=4 cm when t=2 seconds ; find the distance when t=5 seconds and the acceleration of the particle at the same time. [5 marks]

QUESTION THREE (20 MARKS)

a. State **THREE** uses of dimensional analysis [3 marks]

b. A particle moves in a circle of radius 20 m. if its tangential speed is 40m/s. find

   i. The angular speed [2 marks]

   ii. The angular and normal accelerations [4 marks]

c. Given $\emptyset = 2x^3y^2z^4$

   i. Find $\nabla.\nabla\emptyset$ [3 marks]

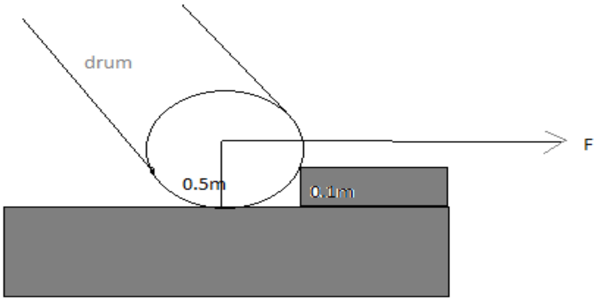
   ii. Show that $\nabla.\nabla\emptyset = \nabla^2$ where

   $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is called the Laplacian Operator [3 marks]

d. Two particles have position vectors given by $r_1 = 4ti - 2t^2 - 5tk$ and $r_2 = (2t^2 - t)i + t^3j - 4tk$. Find the relative velocity and acceleration of the second particle with to the first when t=3 seconds [5 marks]

QUESTION FOUR (20 MARKS)

a. Figure below shows a drum of mass 150kg and radius 0.5m being pulled by a horizontal force F against a step 0.1m high. What initial force is just sufficient to turn the drum so that it raises over the step [4 marks]



b. Find the work done in moving a particle once around a circle c in the x-y plane with center origin and radius three units by a force given by

$$F = (2x - y + z)i + (x + y - z^2)j + (3x - 2y + 4z)k$$

for $x = 3\cos\Theta$ and $y = 3\sin\Theta$. With θ changing from zero to $360^0(2\Pi)$

[6 marks]

c. A particle P projected from a point O on a horizontal plane with a speed of 72km/h at an angle Θ to the horizontal, where $\tan\Theta = \frac{4}{3}$. find

i. Time taken for P to return to the plane [3 marks]

ii. Maximum time reached [2 marks]

iii. The range [2 marks]

iv. Speed after 2 seconds [3 marks]

QUESTION FIVE (20 MARKS)

a. Derive the dimensional expression for the second equation of motion [3 marks]

b. A coil spring lies along the helix $r = (cos4t)i + (sin4t)j + tk,\ 0 \leq t \leq 2\Pi$. The spring's density is a constant, δ=1.find the springs mass, the coordinates of the spring's centre of mass and the spring's moment of inertial and radius of gyration about Z axis. [7 marks]

c. If $\emptyset(x, y, z) = 3x^2y - y^3z^2$ find $\nabla\emptyset$ at (1, -2,-1) [4 marks]

d. A lorry of mass 200kg moving at10m/s on a horizontal surface is brought to rest in a distance of 12.5m by the brakes being applied.

i. Calculate the average retarding force F [3 marks]

ii. What power must the engine produce if the lorry is to travel up a hill of 1 in 10 minutes at a constant speed 10m/s; if friction resistance is 200N. [3 marks]

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