Technical University of Mombasa

AMA: 4102 / SMA: 2171 / SMA: 2177 - GEOMETRY

UNIVERSITY EXAMINATION FOR THE FIRST SEMESTER IN THE FIRST YEAR OF BACHELOR OF SCIENCE IN STATISTICS, MATHEMATICS AND COMPUTER SCIENCE, MECHANICAL, CIVIL, ELECTRICAL AND ELECTRONICS, MEDICAL ENGINEERING.

**DECEMBER 2015: EXAMINATION**   **TIME: 2HOURS**

Answer Question One And Any Other Two Questions.

Question One (30 marks) compulsory.

a) Find the shortest distance from the point (2, 3) to the line 5x-12y=52.   2 Marks

b) Show that $\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 4\cos\theta\cos 2\theta\cos 4\theta$ 4 Marks

c) Find the equation of an ellipse whose vertices are the points (-1, 2) and (9, 2) while eccentricity is $\frac{2}{3}$.   4 Marks

d) Determine the length of a tangent from the point (5, 7) to the circle whose equation is $x^2 + y^2 - 4x - 6y + 9 = 0$.   4 Marks

e) If sinA=3/5 and cos B=15/17 where A is obtuse and B acute, find the exact value of sin(A+B) .   4 Marks

f) Two sides of an acute angled triangular piece of land are 48m and 26 m respectively. If the area of the plot is $550\,m^2$, find the length of a third side.   4 Marks

g) Evaluate $7\cos\theta + 2\sin\theta = 4$ if $0^0 \le \theta \le 360^0$.   4 Marks

h) Find an equation for a hyperbola with asymptotes $y = \pm\frac{4}{3}x$ and foci $(\pm 10, 0)$.   4 Marks

Question Two (20 marks)

a) A 100N weight hangs from cables as shown below:
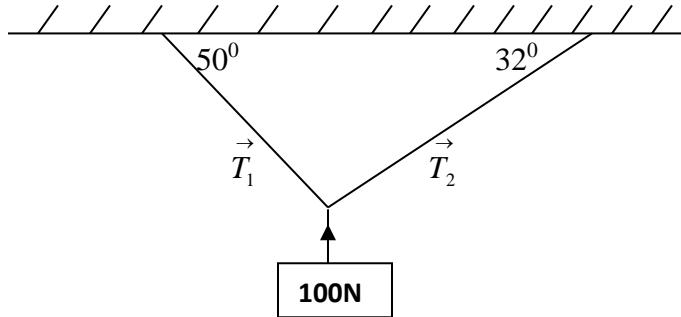
Find the tension forces $\vec{T_1}$ and $\vec{T_2}$.   7 Marks

b) Solve the equation $3\cos 2\theta + \sin \theta = 1$ for values of $0 \le \theta \le 360$.   6 Marks

c) Find the points in which the lines 5y=3x-5 cuts the hyperbola $4x^2 - 25y^2 = 15$ and the equations of the tangent to the hyperbola at these points.   7 Marks

Question Three (20 marks)

a) Plot accurately the graph of the polar equation $r = \sin 2\theta$ and mark the lines of symmetry on the grid. How many lines of symmetry exist in this figure, where $0^c \le \theta \le 2\pi^c$.   5 Marks

b) Solve the equation $12\cos^2 \theta + \sin \theta = 11$ on the domain $0^o \le \theta \le 360^o$.   5 Marks

c) Determine the point(s) of intersection of the line 2y =x+6 and the parabola $y^2 = 8x$ hence find the equations of the tangent and normal lines at these intersection points.   7 Marks

d) Given an equation of a line in the form $\dfrac{x-2}{3} = \dfrac{y-4}{5} = \dfrac{z-7}{2}$ express it in the form

$\vec{r} = \vec{a} + t\vec{u}$ hence determine whether the point (8, 14, 11) lies on the line.   3 Marks

Question Four (20 marks)

a) If $\vec{P} = 2i + 3j + 4k$ and $\vec{Q} = 4i - 3j + 2k$ where i, j, k are unit vectors, determine the angle

between the vectors $\vec{P}$ and $\vec{Q}$.   4 Marks

b) If $\tan A = \dfrac{3}{4}$ and $\csc B = \dfrac{17}{8}$ where A and B are acute angles, without first determining the

value of the angles evaluate $\dfrac{3\sin A + 2\cos A}{\sec B}$.   4 Marks

c) Find the eccentricity and semi-latus rectum of the ellipse $4x^2 + 3y^2 = 5$ 6 Marks

d) Find the length of the tangent to the circle $x^2 + y^2 - 2x + 4y - 4 = 0$ from the centre

   of the circle $x^2 + y^2 + 6x + 8y = 0$.   6 Marks

Question Five (20 marks).

a) In a triangular lawn the length of two sides and there included angle are a=12m, b=10m and
$\angle c = 30^0$, calculate the radius of the circumcircle just touching the three corners.   4 Marks

b) Find an equation in the form ax+by+c=0 for a line which passes through the point of

   intersection of the lines $x - 3y = 4$ and $3x + y = 2$ being also perpendicular to the

   line $4x - 3y - 7 = 0$.   5 Marks

c) Discus the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ indicating all properties of the hyperbola

   hence sketch the curves showing the asymptotes, foci and vertex .   8 Marks

d) Find the Cartesian equation of the curve in polar form given as $r\cos\left(\theta - \frac{\pi}{3}\right) = 3$ 3 Marks

THE END