## TECHNICAL UNIVERSITY OF MOMBASA

# Faculty of applied and Health Sciences <br> DEPARTMENT OF MATHEMATICS AND PHYSICS <br> <br> UNIVERSITY EXAMINATION FOR DEGREE IN: <br> <br> UNIVERSITY EXAMINATION FOR DEGREE IN: BACHELOR OF SCIENCE IN CIVIL AND MEDICAL ENGINEERING <br> <br> SMA 2471: NUMERICAL ANALYSIS <br> <br> SMA 2471: NUMERICAL ANALYSIS <br> SPECIAL/ SUPPLIMENTARY EXAMINATIONS <br> SERIES: September 2018 <br> TIME: 2 HOURS 

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of 5 questions. Question one is compulsory. Answer any other two questions Do not write on the question paper.

## QUESTION ONE (30 marks)

a) Prove that $\left(\frac{\Delta^{2}}{E}\right) e^{x} \cdot \frac{E e^{x}}{\Delta^{2} e^{x}}=e^{x}$ the interval of differencing being $h$
(6 marks)
b) Given $f(2)=4, \quad f(2.5)=5.5$ Find the linear interpolating polynomial using Aitkens iteration method and use it to find an approximate value of $f(2.2)$
(6 marks)
c) Using the data $\sin (0.1)=0.09983$ and $\sin (0.2)=0.19867$ Find an approximate Value of $\sin (0.15)$ by Lagrange interpolation.
(5 marks)
d) Given the following values of $f(x)=\log x$, find the approximate value of $f^{\prime}(2.0)$ using the method based on quadratic interpolation.

| $x$ | 2.0 | 2.2 | 2.6 |
| :---: | :--- | :--- | :--- |
| $\log x$ | 0.69315 | 0.78846 | 0.95551 |

e) Find the by Taylors series method to degree three the value of $y$ at $x=0.1$ correct to 5 decimal places from the differential equation $\frac{d y}{d x}=x^{2} y-1, \quad y(0)=1$
(6 marks)
f) Define interpolation

## QUESTION TWO (20 marks)

a) Given that $f(0)=1, f(1)=3, f(3)=5.5$. Find the unique polynomial of degree 2 or less which fits the given data. Hence evaluate the polynomial at 2.5 using Lagrange's fundamental polynomials.
b) Solve the difference equation $\left(\Delta^{2}-3 \Delta+2\right) y_{x}=0$

## QUESTION THREE (20 marks)

a) A slider in a machine moves along a fixed straight rod. Its distance $x \mathrm{~cm}$ along the rod is given below for various values of the time $t$ seconds. Find the velocity and acceleration of the slider when $\mathrm{t}=0.3$ seconds.

| $\mathbf{t}$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x}$ | 30.13 | 31.62 | 32.87 | 33.64 | 33.95 | 33.81 | 33.24 |

b) Given $\frac{d y}{d x}=x^{2}(1+y)$ and $\mathrm{y}(1)=1, \mathrm{y}(1.2)=1.548, \mathrm{y}(1.3)=1.979$, evaluate $\mathrm{y}(1.4)$ using Milne's Predictor-Corrector method.
(10 marks)

## QUESTION FOUR (20 marks)

a) Given the differential equation $\frac{d y}{d x}=x-y$ with the condition $y(0)=1$. Use Picard's method to obtain $y$ for $x=0.2$ up to degree five correct to five places of decimal.
b) Obtain the exact value for (a) above at $x=0.2$

## QUESTION FIVE (20 marks)

a) Given $\frac{d y}{d x}=x+z, \frac{d z}{d x}=x-y^{2}$ with $y(0)=2, z(0)=1$. Obtain the first Taylors algorithm $y_{1}, z_{1}$ for $y(0.1), y(0.2)$
b) Given $f(2)=4, \quad f(2.5)=5.5$. Find the linear interpolating polynomial using Newton's divided difference method and use it to find an approximate value of $f(2.4)$

