# TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR: MECHANICAL AND PRODUCTION ENGINEERING**

**SMA 2371: PDE**

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

**SERIES: September 2018**

**TIME:** TWO HOURS

**DATE: September 2018**

**Instructions to Candidates**
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of FIVE questions. Attempt QUESTION ONE AND ANY OTHER TWO QUESTIONS
**Do not write on the question paper.**

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**Question ONE**

a) Find the general solution of the semi-linear equation $y^2\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} = x(z-2y)$ (5mks)

b) Verify that $u = f(x-ct) + g(x+ct)$ is a solution of the one dimensional wave equation $u_{tt} = c^2 u_{xx}$ (5mks)

c) Find the PDE by eliminating the arbitrary constants $z = (x-a)^2 + (y-b)^2$ and state the order of the resulting PDE (5mks)

d) Show that the direction cosines of the tangent at the point $(x, y, z)$ to the conic $px^2 + qy^2 + rz^2 = 1, x + y + z = 1$ are proportional to $(qy - rz, rz - px, px - qy)$ (4mks)

e) Find the integral curves of the equations $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ (5mks)

f) Solve the equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ using the method of separation of variables (6mks)

**Question TWO**

a) The ends A and B of a rod 20cm long have the temperatures at $30^0 c$ and at $80^0 c$ until steady state prevails. The temperature of the ends are changed to $40^0 c$ and $60^0 c$ respectively. Find the temperature distribution in the rod at time $t$ (13mks)

b) Find the surface which is orthogonal to the one-parameter system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circles $x^2 + y^2 = 1; z = 1$ (7mks)

**Question THREE**

a) Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$ (6mks)

b) Solve the boundary-value problem $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, u(x,0) = 6e^{-3x}$ by the method of separation of variables (6mks)

c) Find the orthogonal trajectories on the surface $x^2 + y^2 + 2fyz + d = 0$ of its curves of intersection with planes parallel to the $x-y$ plane (8mks)

**Question FOUR**

a) Form the PDE by eliminating the arbitrary function from $z = f(x^2 - y^2)$ (4mks)

b) Find the Laplace transform of the function $f(x) = e^{-ax^2}$ (8mks)

c) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ (8mks)

**Question FIVE**

a) Find the integral curves of the equations $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$ (8mks)

b) A string of length $L$ is stretched between points $(0,0)$ and $(L,0)$ on the $x$ axis .At time $t = 0$ it has a shape given by $f(x), 0 \le x \le L$ and it is released from rest .Find the displacement of the string at any latter time (12mks)