TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN:

BACHELOR OF SCIENCE IN CIVIL ENGINEERING/ BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING/ BACHELOR OF SCIENCE IN ELECTRICAL ENGINEERING

SMA 2370: CALCULUS 1V

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

TIME: 2 HOURS

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

QUESTION ONE-30 marks

(a) Express $\frac{dw}{dt}$ as a function of t if $w = xy + z,\ x = \cos t,\ y = \sin t,\ z = t$ (5 marks)

(b) Prove that $\nabla(F+G) = \nabla F + \nabla G$ (5 marks)

(c) Find the Taylors Polynomial $f_n(x)$ for the function $y = e^{2x}$ at $x = 0$ for $n = 4$ (5 marks)

(d) Test whether the mean value theorem holds for the function $f(x) = x - x^3$ on the interval $(-2,1)$ and find the appropriate intermediate value (5 marks)

(e) Find the tangent plane to the surface $z = e^{x^2 - y^2}$ at $(-1,0,e)$ (5 marks)

(f) Evaluate the double iterated integral $\int_{1}^{3}\int_{x^{\frac{1}{3}}}^{x}\frac{y^2}{x}dydx$ (5 marks)

QUESTION TWO-20 marks

(a) If $\vec{A}=(3x^2+6y)i-14yzj+20xz^2k$. Evaluate $\int_c \vec{A}.\vec{dr}$ from (0,0,0) to (1,1,1) along the path $x=t,\ y=t^2,\ z=t^3$ (8 marks)

(b) Show that the Greens Theorem is true for the integral $\oint_c(-ydx+xdy)$ where c is the closed half circle path $-1<x<1$ and $y=\sqrt{1-x^2}$ (12 marks)

QUESTION THREE-20 marks

(a) Evaluate the improper integral $\int_1^{\infty}(1-x)e^{-x}dx$ (8 marks)

(b) Find and classify all critical points of the $f(x,y)=x^3+y^3-3x-12y+20$ (12 marks)

QUESTION FOUR-20 marks

(a) Evaluate $\iint_R x^2-xy+y^2dA$ where R is the ellipse given by $x^2-xy+y^2=2$ and using the transformation $x=\sqrt{2}u-\sqrt{\frac{2}{3}}v,\quad y=\sqrt{2}u+\sqrt{\frac{2}{3}}v$ (12 marks)

(b) Find the equation of the plane tangent to the surface $3x^2+y^2-z^2=-20$ at the point $p_o(1,2,3)$ (8 marks)

QUESTION FIVE-20 marks

(a) Find $\nabla\phi$ if $\phi=\log|\vec{r}|$ (10 marks)

(b) Evaluate the iterated triple integral $\int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 6xydzdydx$ (10 marks)