TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

UNIVERSITY EXAMINATION FOR THE FIRST SEMESTER IN THE SECOND YEAR OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY
ICS 2211: NUMERICAL LINEAR ALGEBRA

SPECIAL/ SUPPLIMENTARY EXAMINATIONS
SERIES: SEPTEMBER 2018

TIME: 2HOURS

DATE: Pick Date **SEPTEMBER** 2018

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.

QUESTION ONE (30 MARKS)

(a) Find the rank of $B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 3 & 2 \\ 2 & 3 & 2 \\ 4 & 3 & 2 \end{pmatrix}$. (3mks)

(b) Describe the term ‘the transpose of a matrix’. Hence verify that $(AB)^T = B^T A^T$ if $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ -1 & 5 \end{pmatrix}$. (4mks)

(c) If $A=\begin{pmatrix} x & 3 \\ 2 & 2x+1 \end{pmatrix}$, find the value(s) of x for which A is singular. (4mks)

(d) Compute AB and BA if possible given that

$$A=\begin{pmatrix} 1 & -3 & 0 \\ -2 & 5 & -8 \end{pmatrix},\ C=\begin{pmatrix} 8 & 5 & 3 \\ -3 & 10 & 2 \\ 2 & 0 & -4 \end{pmatrix}$$. Hence make any relevant conclusion.

(4marks)

(e) Find the inverse of

$$B=\begin{pmatrix} 3 & 4 & 5 \\ 1 & -1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$$

by reducing the augmented matrix [B/I] to canonical form. (5mks)

(f) Find the eigenvalues and the corresponding eigenvectors of the matrix $A=\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$ (5mks)

(g) Consider the linear system

$$kx+y+z=1$$
$$x+ky+z=1$$
$$x+y+kz=1$$

For what values of k does the system have no solution? (5mks)

QUESTION TWO (20 MARKS)

(a) Use elimination method to solve the following system of linear equations

$$2x-4y+6z=20$$
$$3x-6y+z=22$$
$$-2x+5y-2z=-18$$

(6mks)

(b) Use Cramer's rule to solve

(7mks)

$$3x-y+z=5$$
$$2x+2y+3z=4$$
$$x+3y-z=11$$

(c) Use Gauss-Jordan elimination method to solve

$$w+x+y+z=4$$
$$w+2x-y-z=7$$
$$2w-x-y-z=8$$
$$w-x+2y-2z=-7$$

(7mks)

QUESTION THREE (20 MARKS)

(a) Find the value of t such that the determinant of $A=\begin{pmatrix} t+3 & 0 & 0 \\ 5 & t-3 & 1 \\ 6 & -6 & t+4 \end{pmatrix}$ is zero. (4mks)

(b) Given that $3\begin{pmatrix} x & y \\ z & w \end{pmatrix}=\begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix}+\begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}$, find the values of x,y,z and w.

(5mks)

(c) If $A=\begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$. (5mks)

(d) If $A=\begin{pmatrix} 6 & -2 & 0 \\ 9 & -1 & 1 \\ 3 & 7 & 5 \end{pmatrix}$ find the LU-decomposition of A. (6mks)

QUESTION FOUR (20 MARKS)

(a) Use inverse matrix method to solve the system

$$x+2y+3z=5$$
$$2x+5y+3z=3$$
$$x+8z=17$$

(9mks)

(a) Use Jacobi iterative method to find the approximate solution for the following system correct to 4 d.p Take the initial approximate to be $x_1=0, x_2=0$ and $x_3=0$ for four iterations only.

$$20x_1-x_2+x_3=20$$
$$2x_1+10x_2-x_3=11$$
$$x_1+x_2-20x_3=-18$$

(11mks)

QUESTION FIVE (20 MARKS)

(a) The solution to a system of equations having the form $AX=B$ can be found by matrix multiplication.

$$X=\begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$$

(i) Find the matrix A. (5mks)

(ii) Find the original system of equations. (3mks)

(iii) Find the solution of the system. (3mks)

(b) Solve the following system by Gauss- Seidel iteration. Approximate the solution to *4d.p*. Take the initial approximate to be $x_1 = 0, x_2 = 0$ and $x_3 = 0$ for three iteratives only.

$$10x_1 + x_2 + 2x_3 = 3$$
$$x_1 + 10x_2 - x_3 = 1.5$$
$$2x_1 + x_2 + 10x_3 = -9$$

(9mks)