TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATIONS 2017/2018
APS 4401: ELECTRODYNAMICS

SERIES: FEBRUARY 2018 TIME: 2 HOURS

Instructions to candidates:
You should have the following for this examination
Answer booklet, Examination paper, Examination Pass and Student ID.
1. This examination paper contains Five Questions:
   Question **ONE** carries **30 marks** while the rest of the questions carry **20 marks** each.
2. Answer question **ONE** and any **TWO** of the other questions.

QUESTION ONE (30 Marks)

a) Distinguish between the permittivity of free space and permeability of free space (4 marks)
b) Define the curl of a vector (2 marks)
c) Calculate the divergence of the vector function
$$\mathbf{v} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$$ (3 marks)
d) State Gauss's law (2 marks)
e) Derive an expression of the Gauss's law in differential form (6 marks)
f) Distinguish between surface and volume charge densities (2 marks)
g) A current I is uniformly distributed over a wire of circular cross section, with radius a.
   i. Find the volume current density J. (2 marks)
   ii. Suppose the current density in the wire is proportional to the distance from the axis, $J = ks$ for some constant k, find the total current in the wire. (4 marks)
h) Find the magnetic field at a distance s from an infinitely long straight wire carrying a steady current I. (5 marks)

QUESTION TWO (20 Marks)

a) Show that the curl of a gradient function is always zero (3 marks)
b) State the Green's theorem and its physical interpretation (3 marks)
c) From the simple configuration of a point charge at the origin, show that
$$\oint \mathbf{E} \cdot \mathbf{dl} = 0$$
and hence $\nabla \times \mathbf{E} = 0$ (10 marks)

d) Use Gauss's law to explain the discontinuity of the electric field vector while crossing a surface charge. (4 marks)

QUESTION THREE. (20 Marks)

a) State Helmholtz theorem (3 marks)
b) State the equivalent conditions for a divergenceless (or solenoidal) fields (4 marks)
c) Derive the equation of continuity $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ (6 marks)
d) Show that the integral of the magnetic field around a circular path of radius s, centered at the wire, is $\oint \mathbf{B} \cdot \mathbf{dl} = \mu_0 I$. (7 marks)

QUESTION FOUR. (20 Marks)

a) State the general expression of a vector field in terms of potentials (2 marks)
b) Discuss the lack of uniqueness property of potentials in the theory of vector fields (4 marks)
c) Let $\mathbf{F}_1 = x^2\hat{z}$ and $\mathbf{F}_2 = x\hat{x} + y\hat{y} + z\hat{z}$ be vector fields
   i. Which one can be written as the gradient of a scalar? (1 mark)
   ii. Find a scalar potential that does the job (3 marks)
   iii. Which one can be written as the curl of a vector? (1 mark)
   iv. Find a suitable vector potential (3 marks)
d) Suppose $\mathbf{V} = (2xz + 3y^2)\hat{y} + 4yz^2\hat{z}$. Check the Stoke's theorem for a square surface of side 1 unit (6 marks)

QUESTION FIVE (20 Marks)

a) State Maxwell's equations related to the divergence and curl of the electric field vector (4 marks)
b) Show that the differential form of Ampere's law for time-varying field is
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ (10 marks)
c) What is the physical interpretation of equation (b) above (4 marks)
d) Define displacement current (2 marks)