TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY (BTRE &BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

APS 4307: MATHEMATICAL PHYSICS.

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME:2HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of 4 questions. Attempt question 1 and any other two questions

Do not write on the question paper.

Question 1 [30 marks]

a) Show that if $f(p)$ is the Laplace transform of $f(x)$ i. e. $f(p) = L\{f(x)\}$, then the Laplace transform of

$\frac{f(x)}{x}$ is given by $\frac{d}{dp}\left(L\left\{\frac{f(x)}{x}\right\}\right) = -f(p)$ [4 marks]

Hence determine the Laplace transform for the functions

ii. $f(x) = \frac{e^{-a/x}}{x^{1/2}}$ [5 marks]

and

iii. $f(x) = \frac{e^{-a/x}}{x^{3/2}}$ [3 marks]

b) Express the following integral in terms of gamma function(s)

$$\int_0^\infty e^{-x^3} x^5 dx$$ [4 marks]

c) Show that the following set of matrices form a representation of the cyclic group $C_4 = (A, A^2, A^3, A^4 = E)$

$$D(A) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, D(A^2) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, D(A^3) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \text{ and } D(E) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

[5 marks]

d) Interpolate the value of $y = e^{-x^2}$ for the value of $x = 0.2862$ using the Table 1 below

x	$y = e^{-x^2}$	Δ	Δ^2	Δ^3	Δ^4
0	1.00000				
0.05	0.99750	-250			
0.10	0.99005	-745	-495		
0.15	0.97775	-1230	-485	+10	
0.20	0.960679	-1696	-486	+19	+9
0.25	0.93941	-2138	-442	+24	+5
0.30	0.91393	-2548	-410	+32	+8

[5 marks]

e) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y = e^{-x^2}$ at the point $x = 0.05$ from the data of Table 1 above [4marks]

Question 2 [20 marks]

a) Find the inverse Laplace transform of $F(P) = \frac{2}{p^4} + \frac{3}{p^2 + 4}$ [5marks]

f) Evaluate the integral $f(x)=\int_0^\infty \frac{\sin xt}{t}dt$ using Laplace transform [5 marks]

g) Solve $y'' + 2y' + y = e^{-x}\sin x$ using Laplace transform with $y(0)=0$ and $y'(0)=3$

[10 marks]

Question 3 [20 marks]

a) A sphere of radius a is centered at O. It is cut into two equal halves by the x-y plane. The upper part is maintained at a potential $+V_0$ and the lower part at potential $-V_0$. Calculate the potential at a point inside the sphere int the following steps:

i. Write the Laplace's equation satisfied by the potential in spherical coordinates and make use of separation of variables to separate it into the $\varphi-, \theta-,$ and $r-$equations [4marks]

ii. Solve the $\varphi-, \theta-,$ and $r-$equations [5 marks]

iii. Make use of the boundary conditions to find the potential [5marks]

b) Using the table given below, evaluate the integral $\int_0^1 \frac{x^3}{e^x-1}dx$ using Simpson's one third rule. [6marks]

x	$f(x)=\frac{x^3}{e^x-1}$
0	0
0.25	0.055013
0.50	0.192687
0.75	0.377686
1.00	0.581977
1.25	0.784280
1.50	0.969357

Question 4 [20 marks]

a) Define the following

i. Subgroup [2 marks]

ii. Isomorphism [2marks]

iii. Unitary matrix [2marks]

iv. Hermitian matrix [2marks]

v. A faithful representation [2marks]

b.Show that the set of all positive and negative integers including zero form a group under addition [5marks]

Question 5 [20 marks]

a) Construct the Green's function for the problem stated mathematically as $\frac{d^2y}{dx^2}+\omega^2 y=f(x)$ where $f(x)$ is a known function and y satisfies the boundary conditions $y(0)=0$ and $y(L)=0$ [10marks]

b) Determine the Laplace transforms for

i. The error function [6 marks]

ii. The complimentary error function [4marks]