TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATIONS 2017/2018
APS 4302: QUANTUM MECHANICS I

SERIES: SEPTEMBER 2018 TIME: 2 HOURS

Instructions to candidates:
You should have the following for this examination
Answer booklet, Examination paper, Examination Pass and Student ID.

1. This examination paper contains Five Questions:
   Question **ONE** carries **30 marks** while the rest of the questions carry **20 marks** each.
2. Answer question **ONE** and any **TWO** of the other questions.

QUESTION ONE (30 Marks)

a) Briefly describe quantum mechanics and state its importance (4 marks)

b) Explain how the concept of black body radiation led to the development of quantum mechanics (6 marks)

c) Outline the fundamental contributions of Schrödinger and Born to quantum mechanics as a discipline (4 marks)

d) Starting with the description of a plane monochromatic wave, show that the energy of a quantum system can be represented using the operator

$$E \to i\hbar \frac{\partial}{\partial t}$$ (6 marks)

e) Show that the time-independent Schrödinger equation for a free particle in one-dimension is given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x) = E\Psi(x)$$ (6 marks)

f) State a precise three-dimensional generalization of the equation in (e) above for a bound system (4 marks)

QUESTION TWO (20 Marks)

a) Define the following terms:
   i. Eigenvector and eigenvalue (2 marks)
   ii. Quantum operators (2 marks)
   iii. Stationary states (2 marks)

b) Show that the general solution for a one-dimensional time-dependent Schrödinger equation for a stationary state can be expressed as

$$\Psi(x,t) = \sum_n a_n \psi_n(x) \exp(-iE_n t)$$ (14 marks)

QUESTION THREE (20 Marks)

a) Explain the meaning of the following terms:
   i. Degeneracy (2 marks)
   ii. Parity (2 marks)

b) Solve the Schrödinger equation for a particle of mass μ in a $1-D$ infinitely deep potential well characterized by the potential energy variation of the form

$$V(x)=\begin{cases}0 & \text{for } 0<x<a\\ \infty & \text{for } x<0 \text{ and } x>a\end{cases}$$ (16 marks)

QUESTION FOUR (20 Marks)

a) Write the expectation values of the position and momentum co-ordinates of a quantum system. What is the physical interpretation of these expectation values? (4 marks)

b) Consider the potential step for which

$$V(x)=\begin{cases}0 & \text{for } x<0\\ V_0 & \text{for } x>0\end{cases}$$

Show that for $E<V_0$, where E the total energy of the particle, there is a finite probability of finding the particle in the region $x>0$. (16 marks)

QUESTION FIVE (20Marks)

a) State the boundary and continuity conditions of the wavefunctions (4 marks)

b) Calculate the reflection and transmission coefficients for a rectangular potential barrier given by

$$V(x)=\begin{cases}0 & \text{for } x<0\\ V_0 & \text{for } 0<x<a\\ 0 & \text{for } x>a\end{cases}$$

Consider the incident wavefunction to be of the form $\Psi(x)=A\exp(ikx)$ and the condition $E<V_0$. (16 marks)