TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

SEPTEMBER 2018 SERIES EXAMINATION

EMG 2414: NUMERICAL METHODS FOR ENGINEERS

BSME/BEME

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. A production company produces depending on demand. The production of a special component shall follow the demand below. Find the average demand per day over 10 days period based on

Demand per day	4	5	6	7	8
probability	0.10	0.15	0.25	0.30	0.20

With **random numbers** 41 92 05 44 66 07 00 00 00 14 62 20 07 95 64 26 06 48 (4 marks)

b. If $AA^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ (2 marks)

c. Using **Romberg's** integration method, find the value if starting with trapezoidal rule for the given tabular values below given $h = 0.8\ for\ \frac{h}{2}, \frac{h}{4}, \frac{h}{8}$ (7 marks)

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8

Y=f(x)	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.8228	3.107

d. Evaluate

$\int_0^1 \frac{1}{1+x} dx$ Using **Trapezoidal** rule for n=1 and state the error bound (4 marks)

e. Find $\lambda \; for \begin{pmatrix} \lambda & \lambda \\ 3 & \lambda - 2 \end{pmatrix} = 0$ (3 marks)

f. Consider the system of equations

$$\frac{dx}{dt} = x - y - z$$
$$\frac{dy}{dt} = y + 3z$$
$$\frac{dz}{dt} = 3y + z$$

i. Write in the normal form
ii. Find Eigen values and Eigen vectors (4 marks)
iii. Prove that the solutions are independent (2 marks)
iv. Hence, obtain a general solution (1 marks)

g. Given

$$\frac{dy}{dt} = \frac{y - t}{y + t}$$

With the initial condition $y = 1$ at $t = 0$

Find an approximate value of y at $x = 0.1$, in five steps using **Euler's** method. (3 marks)

QUESTION TWO (20 MARKS)

a. Define **linear independence** of functions (2 marks)

b. Obtain **Picard's second** approximate solution of the initial value problem $\frac{dy}{dx} = \frac{x^2}{y^2+1}, y(0) = 0$ (4 marks)

c. Find the inverse of the matrix A by **Gaussian** method

$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}$ (6 marks)

d. Solve the initial value problem

$$\frac{dy}{dx} = x^2 - y \qquad y(0) = 1$$

With h=0.2 on the interval (0, 0.4) using the fourth order **Runge-Kutta** Method.

With $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(t_n, y_n)$

$k_2 = hf(t_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = hf(t_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = hf(t_n + h, y_n + k_3)$ (8 marks)

QUESTION THREE (20MARKS)

a. Determine the **ad joint** of a matrix A if

$$A = \begin{pmatrix} 1 & 5 & -2 \\ 3 & -1 & 4 \\ -3 & 6 & -7 \end{pmatrix}$$

Hence compute A^{-1} the inverse of the matrix (6 marks)

b. I) Find the **determinant** of the following matrix

$$\begin{pmatrix} -4 & 9 & 2 \\ 5 & 6 & -1 \\ 3 & 2 & 7 \end{pmatrix}$$ (3 marks)

ii) For the system of equations given below, form the augmented matrix hence solve for the three unknowns using **Gaussian** elimination method (5 marks)

$$2x + 4y + 7z = 82$$
$$6x - 3y + z = 11$$
$$x + 2y - 5z = -27$$

c. Compute the integral $I = \sqrt{\frac{2}{\pi}} \times \int_0^1 e^{\frac{-x^2}{2}}\, dx$ using **Simpson's $\frac{1}{3}$rd rule** taking h=0.125 (6 marks)

QUESTION FOUR (20 MARKS)

a. Using **Crout's** reduction, decompose the matrix A

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 1 & 3 \end{pmatrix}$$

In to$[L][U]$ form and hence solve the system of equations

$x + y + z = 7$

$3x + 5y + 4z = 24$

$3x + y + 3z = 16$ (7 marks)

b. Use **Gauss Legendre quadrature** formula to compute the integral

$I = \int_5^{12} \frac{dx}{x}$ for n=3 in the interval (-1, 1) (5 marks)

c. Given that $A = \begin{pmatrix} 1+j & 2j \\ -3j & 1-4j \end{pmatrix}$ and that $j^2 = -1 = j.j$. Determine det A (2 marks)

d. If A(t) = $\begin{pmatrix} e^t & 2e^{-t} & e^{2t} \\ 2e^t & e^{-t} & -e^{2t} \\ e^t & 3e^{-t} & 2e^t \end{pmatrix}$

i) Find $\frac{dA}{dt}$ (2 marks)

ii) Determine $\int_0^1 A(t)dt$ (4 marks)

QUESTION FIVE (20 MARKS)

a. Employ **Taylors** method to obtain an approximate value of y at x=0.2 for the differential equation given below and compare the results with the exact solution $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ (5 marks)

b. Given $\frac{dy}{dx} = 1 + y^2$ approximate $y(0.8)$ using the **Milne's Predictor** Corrector method if

x	0.0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841

(7 marks)

c. Consider the following system of linear equations.

$$x + y + z = 4$$
$$2x - 3y + 4z = 33$$
$$3x - 2y - 2z = 2$$

Apply **Crammer's rule** to determine x, y and z

(6 marks)

d. Given that $A = \begin{pmatrix} 3 & 2-i \\ 4+3i & -5+2i \end{pmatrix}$

Find the ad joint of A (2 marks)