TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering and Technology
Department of Mechanical & Automotive Engineering
UNIVERSITY EXAMINATION FOR:
BSc. Mechanical Engineering
EMG 2405 : Control Engineering I
SUPPLEMENTARY EXAMINATION
TIME: 2 HOURS

Instruction to Candidates:

You should have the following for this examination

- *Answer booklet*
- *Non-Programmable scientific calculator*

This paper consists of **FIVE** questions. Attempt question **ONE** and any other **TWO** questions.

Maximum marks for each part of a question are as shown.

Do not write on the question paper.

Question ONE (Compulsory)

a. Draw a block-diagram of a typical negative-feedback, closed-loop system, labelling the individual blocks and the system inputs and outputs. State two advantages and two disadvantages of closed-loop systems when compared to open-loop systems.
(5 marks)

b. Consider the electrical circuit shown in Figure Q1(b) which consists of a capacitor in series with a resistor. R is the resistance of the resistor and C is the capacitance of the capacitor. The voltage input to the circuit is $v(t)$ whilst the potential differences across the capacitor and resistor at any time are $v_C(t)$ and $v_R(t)$ respectively.

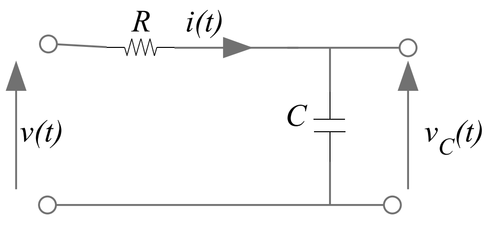


Figure Q1

i. Derive a relationship between the output potential difference across the capacitor, $v_C(t)$, and the input voltage, $v(t)$ **(2 marks)**

ii. Take Laplace transforms of the equation to give an expression for the relationship between, $V_C(s)$ and $V(s)$, the Laplace transforms of $v_C(t)$, and $v(t)$ respectively. Remember that the initial value of $v_C(t)$ may not be zero.
(3 marks)

iii. If the resistance R = 5 kΩ and the capacitance C = 40 μF , calculate an expression for the potential difference across the capacitor, $v_C(t)$, if there is a step input voltage applied to the system at time t = 0 equal to $v(t)$ = 240V. Sketch the overall response of the capacitor between 0 and 0.5 seconds, assuming that the initial value of $v_C(t)$ is zero. This may be done using the trial solution or Laplace transform approach. **(5 marks)**

c. Consider the closed loop system with a unity feedback as shown in Figure Q1b.

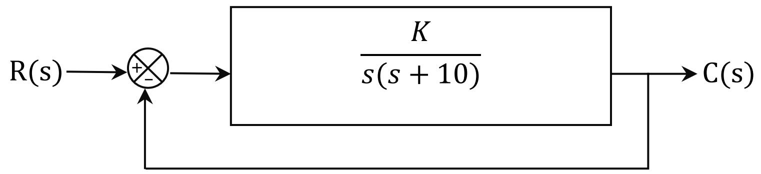


Figure Q1b

i. Determine the gain K so that the system will have a damping ratio of 0.5.

ii. For the obtained value of K determine the following for a unit step input:

i. Settling time,

ii. Rise time,

iii. Time to peak,

iv. Maximum overshoot **(15 marks)**

Question TWO

The plant manager responsible for a chemical plant is unhappy with the performance of one of the processes. When a 10°C reference input step change is applied to the process this only results in a 5°C output temperature change. The time constant of the system is 20 minutes.

a.

i. Assuming that the process can be represented by a first-order model, given the above information, specify the system transfer function, $G_1(s)$, relating the output temperature to the input temperature.

(4 marks)

ii. Sketch the output response of the process to the 10°C reference input step change, clearly indicating the desired steady-state value, the actual steady-state value and the relevance of the time constant. **(8 marks)**

iii. What is the steady-state error of the process? **(4 marks)**

b. Initially, the manager is only concerned with the slow system response to the step change and desires that the time-constant of the system should be reduced from 20 minutes to 5 minutes. The closed-loop system control system is as shown in Figure Q2 where $G_1(s)$ is the transfer function of the process and K(s) is the controller transfer function. The manager decides to use proportional control to achieve this one design requirement. Show that the required value for the proportional controller gain to satisfy the design requirement is K = 6. **(4 marks)**

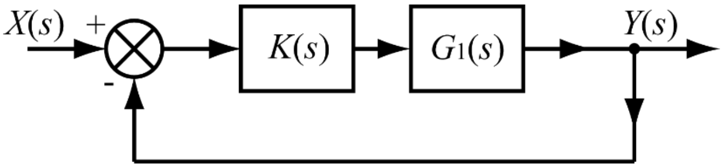


Figure Q2

Question THREE

a. State any TWO advantages of modern control approach to system modeling as compared to classical approach. **(2 marks)**

b. Define the following terms:

   i. State vector,
   ii. State variable, **(2 marks)**

c. Consider a transfer function of a system as shown in Figure Q3c

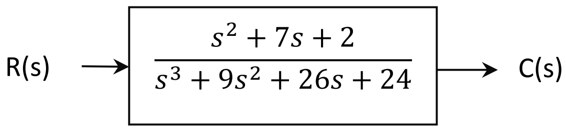


Figure Q3c

   i. Find the state equation and output equation for the phase variable representation of the transfer function.

   ii. Draw an equivalent block diagram showing phase variables. **(10 marks)**

d. Find the equivalent transfer function, *T(s) = C(s)/R(s)*, for the system shown in Figure Q3d

(6 marks)

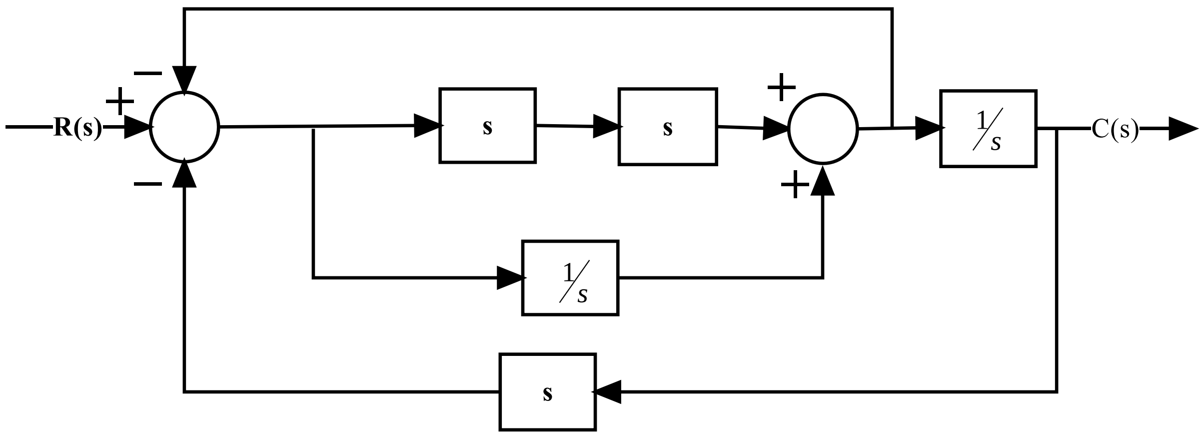


Figure Q3d

Question FOUR

a.

i. State Routh-Hurwitz criteria for stability. **(2 marks)**

ii. Consider the closed loop system shown in Figure Q4a. Determine the range of values of K for which the system is stable. **(8 marks)**

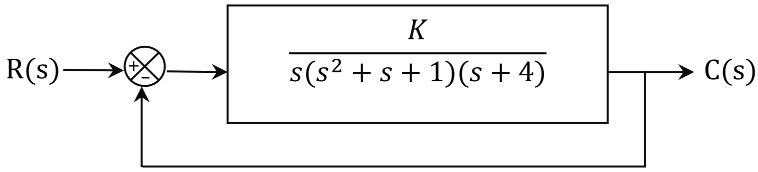


Figure Q4a

b. Consider the mechanical system as shown in Figure Q4a. Determine,

i. The governing differential equation for the system.

ii. State space model for the system. **(10 marks)**

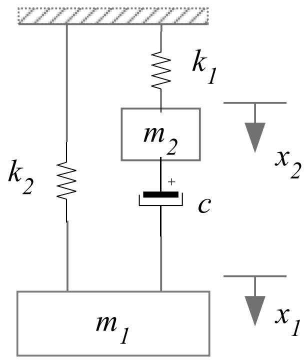


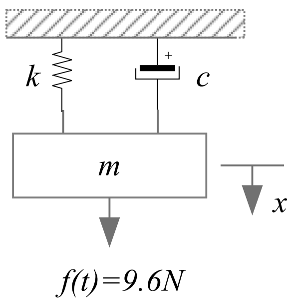
Figure Q4a

Question FIVE

a. Consider a spring-mass-damper system shown in Figure 5a.

i. Determine the transfer function relating the displacement *x(t)* and input force *f(t)* for the spring-mass-damper system.

ii. Obtain the values of c, k and m. The system was initially relaxed. **(20 marks)**



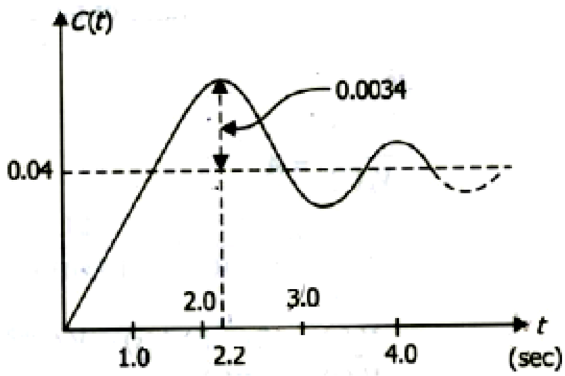


Figure 5a. Figure 5b.

Data Sheet

Laplace Transform Pairs

Time function, $x(t)$	Laplace Transform, $X(s)$
$\delta(t)$, unit impulse at $t=0$	1
1, unit step	$\frac{1}{s}$
t, ramp signal	$\frac{1}{s^2}$
e^{-at}, exponential signal (decaying if $a>0$ growing if $a<0$)	$\frac{1}{(s+a)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\left(\sqrt{1-\zeta^2}\right)t)$; $\zeta<1$	$\frac{\omega_n^2}{\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$; $\zeta<1$
$1-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\left(\sqrt{1-\zeta^2}\right)t+\phi)$ where $\phi=\cos^{-1}\zeta$; $\zeta<1$	$\frac{\omega_n^2}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$; $\zeta<1$
$\frac{B-A\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n\left(\sqrt{1-\zeta^2}\right)t)+$ $A e^{-\zeta\omega_n t}\cos(\omega_n\left(\sqrt{1-\zeta^2}\right)t)$; $\zeta<1$	$\frac{As+B}{\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}$; $\zeta<1$

Laplace Transforms of Systems

$$\mathcal{L}\left\{\frac{dx}{dt}\right\}=sX(s)-x(0) \qquad \mathcal{L}\left\{\frac{d^2x}{dt^2}\right\}=s^2X(s)-sx(0)-\dot{x}(0)$$

$$\mathcal{L}\left\{\int_0^t x(\tau)\,d\tau\right\}=\frac{1}{s}X(s)$$

Final Value Theorem

$$x_{ss}=\lim_{t\to\infty}x(t)=\lim_{s\to 0}sX(s)$$