TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

FOR THE FIRST SEMESTER IN THE FIRST YEAR OF BACHELOR OF SCIENCE IN

MATHEMATICS & COMPUTER SCIENCE, BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE, BACHELOR SCIENCE IN INFORMATION TECHNOLOGY

AMA 4103: CALCULUS I

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME:2HOURS

DATE: Pick DateSep 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **FIVE** questions. Attemptquestion ONE (Compulsory) and any other TWO questions.

Do not write on the question paper.

Question ONE (30 Marks)

a) Let $f:\Re \rightarrow \Re$ be defined by $f(x)=\begin{cases} 2x^3-4, & x \geq 5 \\ 4x-3, & x<5 \end{cases}$ find $f(2),\ f(0),\ f(5)$

(3 marks)

b) Find the derivative of $y=\sqrt{x+3}$ by the first principles (5 marks)

c) Find $\frac{dy}{dx}$ for the following functions

i) $y=x^2 Secx$ (3 marks)

ii) $y=\frac{4xe^{-2x}}{\sqrt{x^2+2x}}$ (4 marks)

d) Evaluate the following limits for x > 0.

(i) $\lim\limits_{x\to 49} \dfrac{x-49}{6-\sqrt{x-13}}$ (4 marks)

(ii) $\lim\limits_{x\to 1} \dfrac{x^3-1}{x-1}$ (3 marks)

e) If $y = xe^{-x}$, show that $\frac{d^2y}{dx^2}$+2$\frac{dy}{dx} + y = 0$ (4 marks)

f) A curve is defined parametrically by $x = t^2, and\ y = \ln(t^2 - 1)$. Find rhe gradient at $t = 2$ (4 marks)

Question TWO(20 marks)

a) Define continuity of a function f at a point $x = a$ (3 marks)

b) (i) Investigate continuity of $f(x)$ at $x = -1$ and $x = 1$ where

$$f(x) = \begin{cases} 2-x, x < -1 \\ x, -1 \le x < 1 \\ 4, x = 1 \\ 4-x, x > 1 \end{cases} .$$ (6maks)

(ii) State, with reasons, whether the function in (i) can be re-defined to be continuous (2marks)

c) Find the equations tangent and normal to the curve $xy + 2x - y = 0$ at (0,0) (9 marks)

Question THREE(20 marks)

a) Let $f : \Re \to \Re$ and $g : \Re \to \Re$, where $\Re$ is the set of real numbers. If $f(x) = x^2 - 2$ and $g(x) = x + 4$ find:

(i) $(f \circ g)(x)$ (2marks)

(ii) $(f \circ g)^{-1}(x)$ (2 marks)

b) Given that $f(0) = 8$, $f'(0) = 3$, $g(0) = 5$, $g'(0) = 1$, find $\dfrac{dy}{dx}$ at $x = 0$ for

$$y = \frac{f(x)}{g(x)} + 3x^2 + 4x$$ (4 marks)

c) Find the $\dfrac{dy}{dx}$ at x= 2 in the following functions

(i) $x = t^3 + t^2,\ \ y = t^2 + t$ (5 marks)

(ii) $y = e^{\sqrt{x^4+1}}$ (5 marks)

(d) Define the limit of a function f at a point x = a (2 marks)

Question FOUR(20 marks)

a) Find the value of k for which the following function is continuous

$$f(x) = \begin{cases} 3x^2 + 1 & , x \leq 1 \\ kx + 1 & , x > 1 \end{cases}$$ (4 marks)

b) Use linear approximation to find, correct to 4 decimal places $y = \sqrt[4]{82}$ (6 marks)

c) (i) Find the stationary points for $y = x^3 + x^2 - 5x - 5$ and classify them (6 marks)

(ii) Sketch the function in (i) (4 marks)

Question FIVE(20 marks)

(a) Differentiate the following function with respect to x

$$y = \ln\sqrt{x^2 - 3}$$ (2 marks)

a) A projectile is fired straight upwards with a velocity of 400m/s. its distance above the ground t- seconds after being fired is given by $S(t) = -16t^2 + 400t$. Find

(i) The time after which the projectile hits the ground. (3 marks)

(ii) The velocity at which the projectile hits the ground (3 marks)

(iii) The maximum altitude achieved by the projectile (4 marks)

(iv) The acceleration at any time t (1 Marks)

b) A rectangular sheet of metal of length 6 metres and width 2 metres is given. Four equal squares of sides x metres are removed from the corners. The sides of this sheet are turned up to form an open rectangular box. Find appropriately, the height of the box which gives the maximum volume. (7 marks)