



TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

SEPTEMBER 2018 SERIES EXAMINATION

AMA 4102 : APPLIED MATHEMATICS 1

EXAMINATION FOR BACHELOR OF TECHNOLOGY IN ELECTRICAL AND ELECTRONICS
ENGINEERING

TIME ALLOWED: 2HOURS

INSTRUCTION TO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

- a. State **Newton's second** law of motion and use it to derive the formula $F = ma$ [3 marks]
- b. Determine the dimensions of E in the **dimensionally** homogeneous Einstein's equation,

$$E = mc^2 \left\{ \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right\}$$

where v is the velocity and m is the mass. [5 marks]

- c. **Derive** the equations of motion for an object projected vertically upwards [5 marks]
- d. A particle is moving along a curve defined by the parametric equation

$$x = 2 \cos 3t \quad y = 2 \sin 3t \quad \text{and} \quad z = 4t^2. \text{ Find}$$

- i. Velocity and acceleration at any given time [3 marks]
- ii. Show that the speed of the particle is increasing but the magnitude of acceleration is constant. [4 marks]
- e. Determine the **unit tangent** T , principal normal N , curvature k and radius of curvature ρ for the space curve
 $x = 3\cos t, y = 3\sin t, z = 4t$ [6 marks]
- f. Find the **total work** done in removing a particle in a force field given by

$$F = 3xyi - 5zj + 10xk \text{ along the curve } x = t^2 + 1, y = 2t^2 \text{ and } z = t^3 \text{ from } t=1 \text{ to } t=2$$

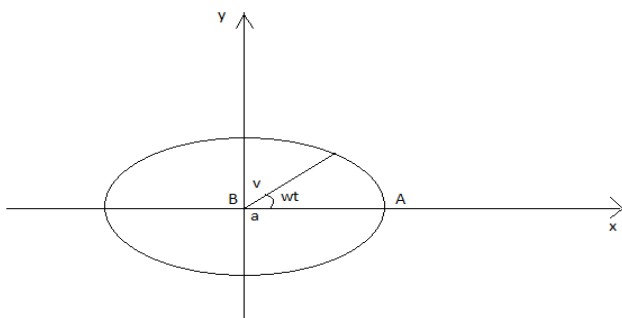
seconds [4 marks]

QUESTION TWO (20 MARKS)

- a. (1) Define a **conservative force** field [1 mark]

(2) A particle of mass m kg moves in the x - y plane so that its position vector $r = a\cos \omega t i + b\sin \omega t j$ where a, b and ω are positive constants and $a > b$

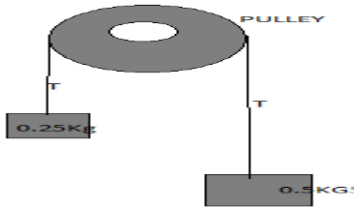
- i. **Show** that the force field is conservative [2 marks]



- ii. Find the **potential energy** at the points A and B in the figure below [4 marks]
- iii. Find the **work done** by force in moving the particle from A to B [2 marks]
- iv. Find the total energy of the particle and show that it is a constant. [2 marks]
- b. A ball of mass 35g travelling horizontally at 20m/s strikes a wall at right angles and bounces with a speed of 16m/s. find the impulse exerted on the ball. [4 marks]
- c. A coin is thrown vertically upwards from the ground with a speed of 10m/s
- How long does it reach the height point [2 marks]
 - What is the maximum height reached by the coin? (3 marks)

QUESTION THREE (20 MARKS)

- a. Find an expression for the **drag force** on a smooth sphere of diameter D , moving with a uniform velocity V in a fluid density ρ and dynamic viscosity μ [8 marks]
- b. Figure below shows Two masses of 0.5 kg and 0.25 kg are connected by a light inextensible string, which passes over a smooth pulley. If the system is released from rest with the string taut, find the acceleration of each mass and the **distance** travelled in 1 second from rest. [6 marks]



- c. A 150 kg mass drum of radius 0.5 m is being pulled by a horizontal force F against a step 0.1 m high. What **initial force** is just sufficient to turn the drum so that it rises over the step. [6 marks]

QUESTION FOUR (20 MARKS)

- a. A projectile is launched with an initial velocity u m/s and at an angle Θ to the horizontal. Determine
- The **time** taken to reach the height point [2 marks]
 - Highest** point reached [3 marks]
 - Time of flight [2 marks]
 - Range [3 marks]
- b. A force given by $6t$ N is acting on a particle whose mass is 12 kg. if it starts from rest determine the **work done** by the force in the first 4 seconds. [4 marks]
- c. A particle whose acceleration is given by $a = 6t^2 + 4t - 1$ has a velocity of 10 m/s when $t=1$ second. Find the **distance** travelled by the particle in the time interval $2 \leq t \leq 10$ [6 marks]

QUESTION FIVE (20 MARKS)

- a. A stone of mass 0.4 kg is tied to a string of length 0.5 m and whirled in a circle. If the stone revolves uniformly and makes one complete **revolution per second**, calculate its acceleration and the force exerted on the stone by the string [4 marks]

- b. Two forces p and q which are inclined at 120° have a **resultant** magnitude of $r = p\sqrt{7}$. Calculate the magnitude of q in terms of p . [6 marks]
- c. A block of mass 2 kg is kept moving with a uniform acceleration of 0.2 m/s^2 by an application of a force of 10.4 N . What is the limiting **frictional force**? [4 marks]
- d. A pilot of a private plane flies 20 km in a direction 60° north of east, then 30 km straight east, then 10 km straight north. How far and in what **direction** is she from the starting point [6 marks]