TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES
DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

DIPLOMA IN MECHANICAL, ELECTRICAL, BUILDING AND CIVIL ENGINEERING

YEAR III SEMESTER II

AMA 2251: ENGINEERING MATHEMATICS IV

SPECIAL/ SUPPLIMENTARY EXAMINATIONS

SERIES: SEPTEMBER 2018

TIME: 2HOURS

DATE: SEPTEMBER 2018

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID Mathematical table, calculator*

This paper consists of **FIVE** questions. Attempt question ONE (Compulsory) and any other TWO questions

Do not write on the question paper.

Question one (compulsory) (30MKS)

a)

i) Determine $L\{e^{4t}\cos 2t\}$ using Laplace transform tables *(3 marks)*

ii) Determine the Laplace transform of $f(t) = 2t$ from first principles

iii) Given that $y = 1$ when $x = 2\frac{1}{6}$, determine the particular solution of $(y^2 + 4)dy = 3ydx$ *(5 Marks)*

iv) Use the integrating factor to solve $\frac{dy}{dx} - \frac{2y}{x} = 3x^3$ given the boundary conditions $y = 1$ when x=3 *(6 Marks)*

v) Determine $L^{-1}\left\{\dfrac{4s-3}{s^2-4s-5}\right\}$ *(4 marks)*

Question Two (20 Marks)

a) Given the function $f(x)=x, \quad 0<x<2\pi$, determine the Fourier series representing the function $f(x)$

(10 Marks)

b) Determine the Fourier series expansion of the periodic function of period 1, given the function

$$f(x)=\begin{cases} -1 & \text{for} \quad -\pi<x<\dfrac{-\pi}{2} \\ 0 & \text{for} \quad \dfrac{-\pi}{2}<x<\dfrac{\pi}{2} \\ 1 & \text{for} \quad \dfrac{\pi}{2}<x<\pi \end{cases}$$

(10 Marks)

<u>Question Three (20 Marks)</u>

a) $\dfrac{dy}{dx}+5y=\dfrac{7}{2}$ and $y=0$ and $x=0$. Use Laplace transforms to the general solution

10 marks)

b) Using Laplace transforms solve the following second order differential equation

$\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+5y=e^{-x}\sin x$, where $y(0)=0$, $y'(0)=1$ (10 Marks)

<u>Question Four (20 Marks)</u>

a) Given that $7x(x-y)dy=2(x^2+6xy-5y^2)dx$ is homogeneous in x and y, solve the differential equation taking $x=1$ when $y=0$ (12 Marks)

b) The charge q in an electric circuit at time t satisfies the equation $L\dfrac{d^2q}{dt^2}+R\dfrac{dq}{dt}+\dfrac{1}{C}q=E$, where L, R, C and E are constants. Solve the equation given $L=2H$, $C=200\times10^{-6}F$ and $E=250V$ when $R=200\Omega$ (8 Marks)

Question Five (20 Marks)

a) Show the following

$e^{3t} = \frac{1}{s-3}$, using the definition of Laplace transforms *(3 marks)*

b) Determine $L^{-1}\left\{\frac{7s+13}{s(s^2+4s+13)}\right\}$

(9 marks)

c) Determine the general solution of $9\frac{d^2y}{dx^2} - 24\frac{dy}{dx} + 16y = 0$, then its particular solution given that $x = 0$, $y = \frac{dy}{dx} = 3$ (8 Marks)

TABLE OF SOME LAPLACE TRANSFORMS		
	Function.	***Transform***
	$F(t)$	$\int_0^\infty e^{-st} f(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/s-a$
3.	$\sin at$	a/s^2+a^2
4.	$\cos at$	s/s^2+a^2
5.	t	$1/s^2$
6.	$t^n (n \ a \ positive \ \text{int} eger)$	$n!/s^{n+1}$
7.	$\sinh at$	a/s^2-a^2
8.	$\cosh at$	s/s^2-a^2
9.	$t \sin at$	$2as/(s^2+a^2)^2$
9(a)	$t \cos at$	$s^2-a^2/(s^2+a^2)^2$
10.	$\sin at - at \cos at$	$2a^3/(s^2+a^2)^2$
11.	$e^{-at}t^n$	$n!/(s+a)^{n+1}$
12.	$e^{-bt}\cos at$	$(s+b)/\left[(s+b)^2+a^2\right]$
13.	$e^{-bt}\sin at$	$a/\left[(s+b)^2+a^2\right]$
14.	$e^{-bt}\cosh at$	$(s+b)/\left[(s+b)^2-a^2\right]$
15.	$e^{-bt}\sinh at$	$a/\left[(s+b)^2-a^2\right]$

	Some Theorems used in Laplace Transforms
1	If $f(s) = \ell\{F(t)\}$ then $f(s+a) = \ell\{e^{-at}F(t)\}$
2	$\ell\left\{\frac{dx}{dt}\right\} = s\,\overline{x} - x_{\circ}$
$\ell\left\{\frac{d^2x}{dt^2}\right\} = s^2\,\overline{x} - s\,x_{\circ} - x_1$	

Table of Laplace Transforms

	$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$		$f(t)=\mathcal{L}^{-1}\{F(s)\}$	$F(s)=\mathcal{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			