TECHNICAL UNIVERSITY OF MOMBASA

Faculty of applied and Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF MATHEMATICS AND COMPUTER SCIENCE

AMA 4213: NUMBER THEORY

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: 2016

PAPER A

<u>Instructions to Candidates</u>

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of 5 questions. Question one is compulsory. Answer any other two questions

Do not write on the question paper.

SECTION A

Question one

(1)(a) Use the axioms of integers to show (i) $0 \bullet a = 0$. (2mks)

(ii) $f\ a < b\ and\ c > 0\ then\ ac < bc$. (2mks)

(b) Show that there is no positive integer less than 1. (2mks)

(c) Evaluate the following

(i) $\sum_{j=1}^{5} 2^{j}$. (2mks)

(ii) $\prod_{j=1}^{5} 2^{j}$. (2mks)

(d) Show that $\sum_{j=m}^{n} ka_j = k\sum_{j=m}^{n} a_j$. (2mks)

(e)(i) Let a,b and c be integers where a/b *and* b/c .Show that a/c . (2mks)

(ii) Let a,b,m and n are integers where c/a *and* c/b . Show that $c/(ma+nb)$.(2mks)

(f) With the use of an example state Division Algorithm. (2mks)

(g) Evaluate (i) (-6,-15) (2mks)

(ii) (0,44). (1mk)

(h) If a and b are integers of the form 4n+1. Show that the product ab is also of the form

4n+1. (2mks)

(i) Show that there are infinitely many primes of the form 4n+3 where n is a positive integer.(3mks)

(j) If $(a,b)=(a,c)=1$. Show that $(a,bc)=1$. (2mks)

(k) (c) State the Wilson's Theorem. (1mk)

(l) Give an example of Fermat number. (1mk)

SECTION B

Question two

(2)(a) Let a,b and c be integers with (a,b)=d. Show that

(i) $\left(a/d, b/d\right)=1.$ (5mks)

(ii) (a+cb,b)=(a,b). (5mks)

(b) Let a and b be non zero integers . Show that the greatest common divisor of a and b is

The least positive integer that is alinear combination of a and b. (6mks)

(c) Find the greatest common divisor for 15,21 and 35. (4mks)

Question three ,(20MKS)

(3)(a) By use of Euclidean algorithm find (252,198). (5mks)

(b) Let n be an odd positive integer. Show that there is one to one correspondence between Factorizations of n into positive integers and differences of two squares that equal n. (6mks)

(c) Use Fermat factorization to factor 6077. (5mks)

(d) The Fermat number $F_5 = 2^{2^5}$ is divisible by 641. Show that $641 / F_5$ without performing actual Division. (5mks)

Question four(20MKS)

(4)(a) Using Fermat numbers show that there are infinitely many primes. (5mks)

(b) For each the following linear Diophantine equations either find all solutions or Show that there are no integral solutions

(i)6x+15y=83. (5mks)

(ii) 20x+50y=510. (7mks)

(c) Test whether the following integers 25 and 42 are relatively prime. (3mks)

QUESTION FIVE (20MKS) .

(5)(a) Let a and b be integers with d=(a,b). Show that

(i) ax+by =c has no integral solutions if d does not divide c. (3mks)

(ii) If d / c there are infinitely many integral solutions. (3mks)

(iii) If $x = x_0, y = y_0$ is a particular solution of the equation then all solutions are given By $x = x_0 + \left(b/d\right)n, y = y_0 - \left(a/d\right)n$ where n is an integer. (2mks)

(b) Let a,b,c and m be integers with m>0 such that $a \cong b(\mathrm{mod}.m)$. Show that

$(i)\ a+c \cong b+c(\bmod .m)$ (3mks)

$(ii)\ a-c \cong b-c(\bmod .m)$ (2mks)

$(iii)\ ac \cong bc \quad (\bmod .m)$ (3mks)

(c) Let a and b be integers. Show that $a \cong b(\bmod .m)$ if and only if there is an integer k such

That a=b+km. (4mks)