TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

DECEMBER 2016 SERIES EXAMINATION

UNIT CODE: AMA 4102 UNIT TITLE: APPLIED MATHEMATICS 1

EXAMINATION FOR BACHELOR OF TECHNOLOGY IN ELECTRICAL AND ELECTRONICS ENGINEERING

SPECIAL EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS) COMPULSORY

a. State Newton's second law of motion and use it to derive the formula $F = ma$ (3 marks)

b. Determine the dimensions of E in the dimensionally homogeneous Einstein's equation,

$$E = mc^2\left\{\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right\}$$

where v is the velocity and m is the mass. (5 marks)

c. Derive the equations of motion for an object projected vertically upwards (5 marks)

d. A particle is moving a long a curve defined by the parametric equation

$x = 2\cos 3t \;\; y = 2\sin 3t$ and $z = 4t^2$. Find

i. Velocity and acceleration at any given time (3 marks)

ii. Show that the speed of the particle is increasing but the magnitude of acceleration is constant. (4 marks)

e. Determine the unit tangent T, principal normal N, curvature k and radius of curvature ρ for the space curve $x = 3cost\ , y = 3sint\ , z = 4t$ (6 marks)

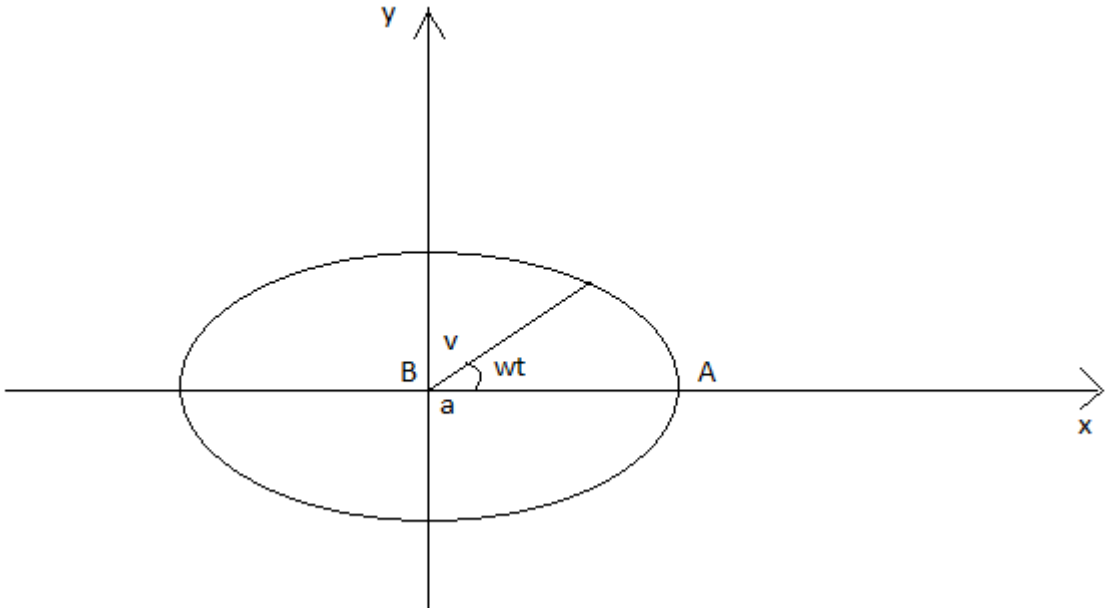
f. Find the total work done in removing a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1,\ y = 2t^2$ and $z = t^3$ from t=1 to t=2 seconds (4 marks)

QUESTION TWO (20 MARKS)

a. (1) Define a conservative force field (1 mark)

(2) A particle of mass m kg moves in the x-y plane so that its position vector $r = a\cos\omega\, ti + bsin\omega\, tj$ where a, b and are positive constants and a>b

i. Show that the force field is conservative (2 marks)



ii. Find the potential energy at the points A and B in the figure below (4 marks)

iii. Find the work done by force in moving the particle from A to B (2 marks)

iv. Find the total energy of the particle and show that it is a constant. (2 marks)

b. A ball of mass 35g travelling horizontally at 20m/s strikes a wall at right angles and bounces with a speed of 16m/s. find the impulse exerted on the ball. (4 marks)

c. A coin is thrown vertically upwards from the ground with a speed of 10m/s

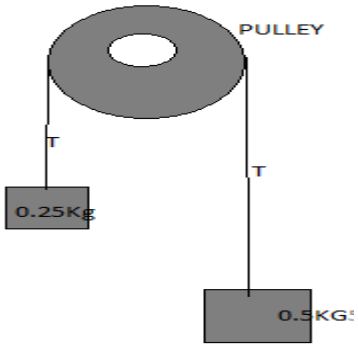
i. How long does it reach the height point (2 marks)

ii. What is the maximum height reached by the coin? (3 marks)

QUESTION THREE (20 MARKS)

a. Find an expression for the drag force on a smooth sphere of diameter D, moving with a uniform velocity V in a fluid density ρ and dynamic viscosity μ (8 marks)

b. Figure below shows Two masses of 0.5 kg and 0.25 kg are connected by a light inextensible string, which passes over a smooth pulley. If the system is released from rest with the string taught, find the acceleration of each mass and the distance travelled in 1 second from rest. (6 marks)



c. A 150 kg mass drum of radius 0.5 m is being pulled by a horizontal force F against a step 0.1 m high. What initial force is just sufficient to turn the drum so that it rises over the step. (6 marks)

QUESTION FOUR (20 MARKS)

a. A projectile is launched with an initial velocity u m/s and at an angle Θ to the horizontal. Determine
   i. The time taken to reach the height point (2 marks)
   ii. Highest point reached (3 marks)
   iii. Time of flight (2 marks)
   iv. Range (3 marks)

b. A force given by 6tN is acting on a particle whose mass is 12 kg. if it starts from rest determine the work done by the force in the first 4 seconds. (4 marks)

c. A particle whose acceleration is given by $a = 6t^2 + 4t - 1$ has a velocity of 10 m/s when t=1 second. Find the distance travelled by the particle in the time interval $2 \leq t \leq 10$ [6 marks]

QUESTION FIVE (20 MARKS)

a. A stone of mass 0.4 kg is tied to a string of length 0.5 m and whirled in a circle. If the stone revolves uniformly and makes one complete revolution per second, calculate its acceleration and the force exerted on the stone by the string (4 marks)

b. Two forces P and Q which are inclined at 120 have a resultant magnitude of $\sqrt{7}$. Calculate the magnitude of Q in terms of P. (6 marks)

c. A block of mass 2 kg is kept moving with a uniform acceleration of $0.2m/s^2$ by an application of a force of 10.4N. What is the limiting frictional force? (4 marks)

d. A pilot of a private plane flies 20 km in a direction 60^0 north of east, then 30 km straight east, then 10 km straight north. How far and in what direction is she from the starting point (6 marks)

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