TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

MAY 2016 SERIES EXAMINATION

UNIT CODE: AMA 4438 UNIT TITLE: APPLICATIONS OF FLUID MECHANICS

SPECIAL/SUPPLIMENTARY EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE (30 MARKS)

a. (i) Distinguish between an ordinary point and a singular point of a differential equation (2 marks)

(ii) Determine the singular points of the equation

$(x^2 - 9)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + (x+3)y = 0$ (4 marks)

b. Define pinch effect in relation to plasma. Hence show that

$p - p_0 = \frac{I\mu}{\pi a^4}(a^2 - R^2)$ Where p is the pressure outside the plasma$R \leq a$ is the radius (6 mks)

c. Find the Laplace transform of $7e^{2t} + 9e^{-2t} + 5cost + 7t^3 + 5sin3t + 2$ (5 marks)

d. Determine the equation of the path of the river flowing that satisfies the equation

$xy\frac{dy}{dx} = x^2 - 1$ and when it passes through a point (1, 2) on the Cartesian plane. (5 marks)

e. State the Hamilton's variation principle for a steady state flow of fluid and define all the symbols you have used. (4 marks)

f. State four ways in which water obtains its ions (4 marks)

QUESTION TWO (20 MARKS)

a. Determine a solution u(x,y) of the Laplace equation $\frac{d^2y}{dx^2} + \frac{d^2y}{dx^2} = 0$ subject to the b.c

$U = 0\ when\ x = 0$ $\quad u = 0\ when\ x = \pi$

$U \to 0\ when\ y \to 0$ $\quad u = 3\ when\ y = 0$ (6 marks)

b. State Luke's Variation principle for a constant equilibrium depth and non constant equilibrium depth for the given 'Ω. outlining the equation of conservation of mass for irrotational flow, the dynamic free surface condition and the condition of zero flow through the bed. (4 marks)

c. Define the following terms according to the knowledge of finite elements

i. The element domain (2 marks)
ii. The space of shape function (2 marks)
iii. A nodal basis (6 marks)

QUESTION THREE (20 MARKS)

a. The efficiency η of a fan depends on the density ρ and the dynamic viscosity μ of the fluid , the angular velocity ω, diameter D of the rotor and the discharge Q. express η in terms of dimensionless parameters (5 marks)

b. Briefly discuss the following aspects ground water flow

   i. Ground water (3 marks)

   ii. Properties of porous media (3 marks)

   iii. Infiltration (3 marks)

c. Locate and classify the singular points of the differential equation

   $(x^2 - 8x)\frac{dy}{dx} + (x + 2)\frac{dy}{dx} + y = 0$ (3 marks)

d. Solve the differential equation below by Laplace transform

   $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 9$ Given that when x=0 y=0 and $\frac{dy}{dx} = 0$ (3 marks)

QUESTION FOUR (20 MARKS)

a. State two factors that affects viscosity (2 marks)

b. A plate 0.05mm distant from a fixed plate moves at 1.2m/s and requires a force of 2.2N/m2 to maintain this speed. Find the viscosity of the fluid between the plate (4 marks)

c. From the Navier Stokes Vector Equation

   $\rho\frac{dy}{dx} = F - \nabla p + \frac{1}{3}\rho v\nabla\nabla . v + \rho v\nabla^2 v$

   Where $\rho =$ fluid density

   $V = fluid\ velocity$

   $F =$ Body force per unit volume

   $P =$ Fluid pressure

   $v =$Kinematic coefficient of viscosity

   $\rho v =$Coefficient of viscosity

   Derive the hydrodynamic equation of motion of a conducting fluid. (7 marks)

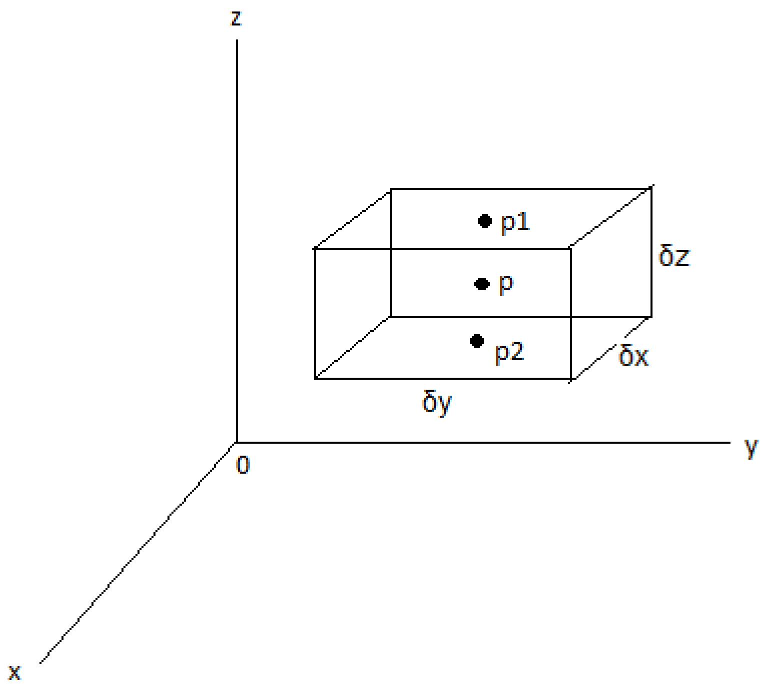
d. Discuss the following

   i. Properties of porous media (4 marks)

   ii. Propreties of water (3 marks)

QUESTION FIVE (20 MARKS)

a. Consider the motion of a small rectangular parallel piped of viscous fluid, its centre being $p(x, y, z)$ and its edges of length $\delta x, \delta y, \delta z$parallel to fixed Cartesian axes.



Taking the mass of the fluid element to be constant and the element to move along the fluid. Show that there is a relationship between the Cartesian components (6 marks)

b. Show that for a conducting fluid at rest , the charge decays very rapidly (6 marks)

c. A coil spring lies along the helix. $r = (cos4t)i + (sin4t)j + tk, 0 \leq t \leq 2\pi$. The spring's density is a constant δ=1. Find spring's mass and spring's moment of inertia and radius of gyration about the z axis (3 marks)

d. State and proof the Alfven's theorem for a perfect conducting fluid (5 marks)

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