TECHNICAL UNIVERSITY OF MOMBASA

A Centre of Excellence

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS AND PHYSICS

JULY 2017 SERIES EXAMINATION

BMCS

UNIT CODE: AMA 4437 UNIT TITLE: CONTINUUM MECHANICS

SPECIAL EXAMINATION

TIME ALLOWED: 2HOURS

INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

QUESTION ONE

a. Prove that $(a.bxc)r = (a.r)bxc + (b.r)cxa + (c.r)axb$ (4 mks)

b. The state of stress at a point is given by the stress tensor

$$\sigma_{ij} = \begin{pmatrix} \sigma & a\sigma & b\sigma \\ a\sigma & \sigma & c\sigma \\ b\sigma & c\sigma & \sigma \end{pmatrix}$$

where a,b,c are constants and σ is some stress value. Determine the constants (5 mks)

c. For the stress having ${\sigma_{ij}}^D = \beta D_{ik} D_{kj}$ determine the dissipation function in terms of the invariants of the rate of deformation tensor D (5 mks)

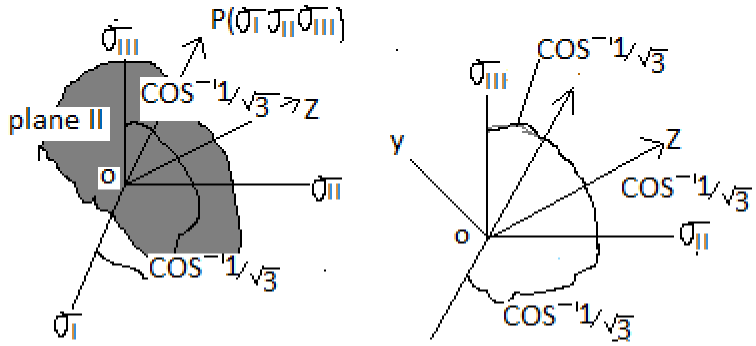
d. Develop the Navier equation for the plane stress and show that it is equivalent to the corresponding equation for plane strain if $\lambda' = \frac{2\lambda\mu}{\lambda+2\mu}$ is substituted for λ (8 mks)

e. Show that for orthotropic elastic continuum (three orthogonal planes of elastic symmery) the elastic coefficient matrix is given by (8 mks)

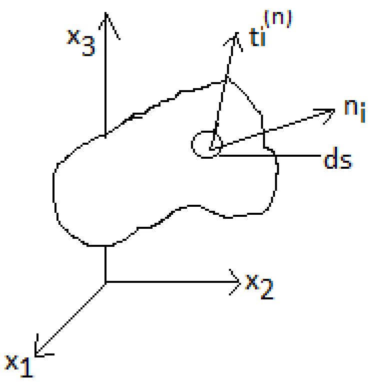
$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}$$

QUESTION TWO

a. With the rectangular coordinate system OXYZ oriented so that the XY plane coincides with the II plane and the σ_{III} axis lies in the YOZ plane as shown in the figure (5 mks)



b. Determine the traction force T acting on the closed surface S which surrounds the volume V of a newtonian fluid for which the bulk viscosity is zero (4 mks)



c. The velocity field is specified by the vector $v = x_1{}^2te_1 + x_2t^2e_2 + x_1x_3te_3$. Determine the velocity and acceleration of the particle at *p(1,3,2)* when t=1. (6 mks)

d. The momentum principal in differential form is expressed by the equation

$$\frac{\partial(pv_i)}{\partial t} = pb_i + \sigma_{ij} - (pv_iv_j),i$$

Show that the equation of motion $\sigma_{ji,j} + \rho b_i = p v_i$ comes from this equation (5 mks)

QUESTION THREE

a. A velocity field is given by $v_1 = 4x_3 - 3x_2,\ v_2 = 3x_1,\ v_3 = -4x_1$. determine the acceleration components at p(b,0,0) and Q(0,4b,3b) and note that the velocity field corresponds to a rigid body rotation of angular velocity 5 about the axis along $e = \frac{4e_2+3e_3}{5}$ (5 mks)

b. For the vectors $a = 3i + 4k\ \ b = 2j - 6k$ and the dyadic $D = 3ii + 2ik - 4j - kj$. Compute by matrix multiplication the products a.D, D.b and a.D.b (5 mks)

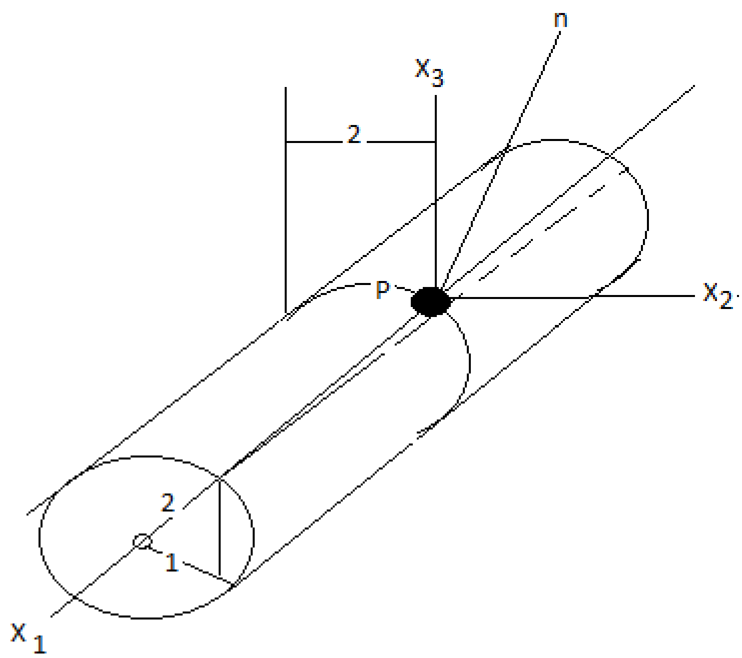
c. (i) For the function $\lambda = A_{ij}x_i x$ where A is a constant, show that $\frac{\partial y}{\partial x} = (A_{ij} + A_{ij})x_i$ and $\frac{\partial y}{\partial x_i \partial x_j} = A_{ij} + A_{ji}$ (3 mks)

(ii)Determine the derivative of the function $\lambda = (x_1)^2 + 2x_1x_2 - (x_3)^2$ in the direction of the unit normal $\boldsymbol{n} = \frac{2e_1-3e_2-6e_3}{7}$ (2 mks)

d. The state of stress throughout the continuum is given with respect to the Cartesian axes $OX_1X_2X_3$ by the array

$$\varepsilon = \begin{pmatrix} 3x_1x_2 & 5x_2 & 0 \\ 5x_2 & 0 & 2x_3 \\ 0 & 2x_3 & 0 \end{pmatrix}$$

determine the stress vector acting at the point P(2,1,3) of the plane that is tangent to the cylindrical surface (5 mks)

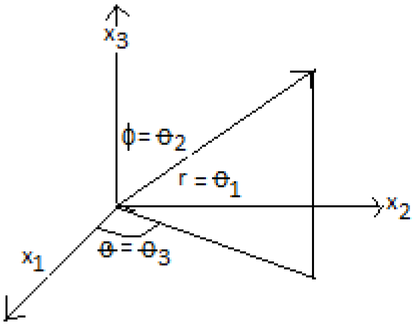


QUESTION FOUR

a. (i) Expand and simplify where possible the expression $d_{ij}x_i xj \quad for \;\; d_{ij} = D_{ji}$ (3 mks)

   (ii) Evaluate the expression involving the kronecker delta for the range of three on the indicies σ_{ii} (2 mks)

b. Determine directly the components of the metric tensor for spherical polar coordinates as shown in the figure (7 mks)



c. The Eulerian description of a continuum motion is given by

$$x_1 = X_1 e^t + X_3(e^t - 1), \quad x_2 = X_3(e^{-t} - e^{-t}) + X_2, \qquad x_3 = X_3$$

Show that the jacobian J does not vanish for this motion and determine the material description by inverting the displacement equation (4 mks)

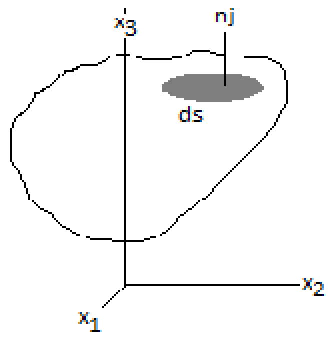
d. Under a load P in a one dimensional test, the true stress is $\sigma = \frac{p}{A}$ while the engineering stress is $s = \frac{p}{A_0}$ where A_0 is the original area and A is the current area. For a constant volume plastic deformation, determine the condition for maximum load. (4 mks)

QUESTION FIVE

a. For the velocity field $v_i = \frac{x_i}{1+t}$, show that $\rho x_1 x_2 x_3 = \rho_0 X_1 X_2 X_3$ (4 mks)

b. In a two dimenstional flow parallel to the $x_1 x_2$ plane, v_3 and $\frac{\partial}{\partial x_3}$ are zero.his case Determine the Navier Stokes equation for an incompressible fluid and the form of the continuity equation for t. (6 mks)

c. Use the divergence theorem of Gauss to show that $\int_S \; x_i n_j ds = d\sigma_{ij}$. where $n_j ds$ represents the surface element of S, the bounding suface of the volume V shown in the figure. (4 mks)



d. The motion of the continuum is given by

$x_1 = Ae^{-\frac{B\lambda}{\lambda}} \sin \lambda(a + \omega t)$

$x_2 = -B - e^{-\frac{B\lambda}{\lambda}} \cos \lambda(A + \omega t)$

$x_3 = X_3$

Show that the particle paths are circles and that the velocity magnitude is constant. Also determine the relationship between X and X and the constants A and B (6 mks)