TECHNICAL UNIVERSITY OF MOMBASA

SCHOOL OF APPLIED AND HEALTH SCIENCES

MATHEMATICS AND PHYSICS

UNIVERSITY EXAMINATION FOR:

UNIT: CONTINUUM MECHANICS

UNIT CODE: AMA 4437

END OF SEMESTER EXAMINATION

SERIES: MAY SERIES

TIME: 2HOURS

<u>**Instructions to Candidates**</u>
You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of five questions. Attempt Question one and any other two.
Do not write on the question paper.

Question ONE

a). Differentiate between Newtonian and Non-Newtonian fluid.   (4mks)

   b). Define the term:

   i.   Plasticity   (2mks)

   ii.   Elasticity   (2mks)

   iii.   Surface forces (Fs)   (2mks)

   c). Discuss the flow for which $w = z^2$ (5mks)

   d). Prove that the contraction of the tensor A_q^p is a scalar or invariant.   (5mks)

e). In an incompressible flow the velocity vector is given by:

$$V= (6xt + yz^2)i + (3t + xy^2)j + (xy\text{-}2xyz\text{-}6tz) \ k$$

Verify whether the continuity equation is satisfied.   (5mks)

f). Work the terms of the indicated sum

$$\overline{g_{rs}} = g_{jk} \ \frac{\partial x^i}{\partial x^{-r}} \ \frac{\partial x^k}{\partial x^{-s}} \qquad\qquad N{=}3 \qquad (5mks)$$

Question TWO

a). If $\emptyset = A \ (x^2 - y^2)$ represent a possible flow phenomena. Determine the stream function.   (4mks)

b). The velocity potential for 2-D flow is

$$\emptyset = x(2y - 1) \ \text{at p(4,5). Determine}$$

   i.   Velocity   (4mks)

   ii.   Value of the stream function   (4mks)

   iii.   Derive the continuity equation   (8mks)

Question THREE

a). Determine the conjugate metric tensor in cylindrical co-ordinates   (7mks)

b). Show that the contraction of the outer multiplication of the tensor A^p and B_q is an invariant.   (6mks)

c). Solve the initial value problem   (7mks)

$$\frac{d^2v}{dt^2} - \frac{2dv}{dt} -8y = 0$$

y (0) = 3

$$y^1(0) = 6$$

Question FOUR

Let T be a second order tensor whose component in the Cartesian System (X_1, X_2, X_3) are given by:-

$$(T)_{ij} = T_{ij} = T = \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Given that the transformation matrix between the system is $(X_1, X_2, X_3) - (X_1^1, X_2^1, X_3^1)$ is

$$A = \begin{vmatrix} 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{vmatrix}$$

a) Obtain the tensor components T_{ij} in the now co-ordinate system (X_1^1, X_2^1, X_3^1) (7mks)

b) The stress state tensor at one point is represented by the carding stress tensor components.

$$\varphi_{ij} = \begin{vmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{vmatrix}$$

Where a, b and c are constants. Determine the constants a, b, c such that the traction vector on the octahedral is the null vector.   (7mks)

c) The carding stress tensor component at the point of a Newtonian fluid, in which the bulk viscosity co-efficient is zero are given by:

$$\varphi_{ij} = \begin{vmatrix} -6 & 2 & -1 \\ 2 & -9 & 4 \\ -1 & 4 & -3 \end{vmatrix} P_a$$

Obtain the viscor's stress tensor component. (6mks)

Question FIVE

Under the restriction of small deformation theory the displacement field is given by

$$\bar{U} = a\,(x_1^2 - -5x_2^2)\,\hat{e}_1 + (2ax_1x_2)\,\hat{e} - (0)\,\hat{e}_3$$

a) Obtain the linear strain tensor and linear spin tensor   (10mks)

b) Given the shear modulus G obtain the value of the young modulus E to guarantee the balance at any point of the continuum.   (10mks)