TECHNICAL UNIVERSITY OF MOMBASA
A Centre of Excellence

DEPARTMENT OF MATHEMATICS AND PHYSICS
JULY 2017 SERIES EXAMINATION
UNIT CODE: AMA 4436

## UNIT TITLE: THEORY OF RELATIVITY

TIME ALLOWED: 2 HOURS

## INSTRUCTIONTO CANDIDATES:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consists of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown

## QUESTION ONE (30 MARKS) COMPULSORY

a. Define relativity
b. State the four equations of Lorentz transformations
c. A hunter on the ground fires a bullet in the northeast direction which strikes a deer 0.25 km from the hunter. The bullet travels at $1800 \mathrm{~km} / \mathrm{hr}$. At the instant when the bullet is fired, an aeroplane is directly over the hunter at an altitude of 1 km and is travelling due east with velocity $600 \mathrm{~km} / \mathrm{hr}$. When the bullet strikes the deer, what are the coordinates as determined by an observer in the plane?
d. By considering two frames of reference $S$ and $\mathrm{S}^{\prime}$ where $\mathrm{S}^{\prime}$ is moving at a speed V . derive the velocity components of Galilean transformations and clearly show the failures of these transformations [6 marks]

## QUESTION TWO [20 MARKS]

a. State FIVE conditions when Doppler effect does NOT take place in sound
b. A submarine travels through water at a speed of $8 \mathrm{~m} / \mathrm{s}$ emitting sonar waves at 1400 Hz . The speed of sound in water is $530 \mathrm{~m} / \mathrm{s}$. a second submarine is located such that both submarines are travelling directly towards each other. The second submarine is moving at $6 \mathrm{~m} / \mathrm{s}$
i. What is the frequency detected by an observer riding in the second submarine as first submarine approaches it?
ii. The submarines barely miss each other and pass. What frequency is detected by an observer riding on second submarine as the subs recede from it?
[2 marks]
iii. When both the submarines approach each other, some of the sound from first submarine reflects from the second submarine and returns to it. If the sound were detected by first submarine, what is its frequency?
c. Suppose an observer S determines that two events are separated by $3.6 \times 10^{8} \mathrm{~m}$ and occur 2 seconds apart. What is the proper time interval between the occurrences of these two events? [5 marks]
d. Find the mass of an electron whose velocity is 0.99C?

## QUESTION THREE [20 MARKS]

a. An airplane travelling at half the speed of sound ( $\mathrm{V}=172 \mathrm{~m} / \mathrm{s}$ ) emits a sound of frequency 5000 Hz . At what frequency does a stationary observer hear the sound
i. As the plane approaches [2 marks]
ii. After the plane passes [2 marks]
b. An astronaut whose height on earth is exactly 6 ft is lying parallel to the axis of a spacecraft moving at 0.9 C relative to the earth. What is his height as measured by
i. An observer in the same plane? [1 mark]
ii. An observer on the earth
c. i. Evaluate $\sqrt{1-\frac{V^{2}}{C^{2}}}$ for $v=10^{-2} C$ and $V=0.9998 C$ [4 marks]
iii. as measured by $S$ a flush bulb goes off at $x=100 \mathrm{~km}, \mathrm{y}=10 \mathrm{~km}, \mathrm{z}=1 \mathrm{~km}$ at $\mathrm{t}=5 \times 10^{-4} \mathrm{~seconds}$.

What are the coordinates $X^{\prime}, Y^{\prime}, Z^{\prime}, t^{\prime}$ of this event as determined by a second observer $S^{\prime}$ moving relative to $S$ at -0.8 C along $X-X^{\prime}$ axis?
d. State the relativistic generalization of Newton's second law of motion
[2 marks]

## QUESTION FOUR [20 MARKS]

a. Derive the formula for time dilation using inverse Lorentz transformation
b. Two observers $A$ and $B$ on a plane whose speed is $2 \times 10^{8} \mathrm{~m} / \mathrm{s}$ both set their watches to the same time. When the spacecraft is abreast the earth, how much time would elapse by A's timing before the watches differ by one second
c. Show that the electromagnetic wave equation $\frac{\partial^{2} \emptyset}{\partial x^{2}}+\frac{\partial^{2} \emptyset}{\partial y^{2}}+\frac{\partial^{2} \emptyset}{\partial z^{2}}-\frac{1}{c^{2}} \frac{\partial^{2} \emptyset}{\partial t^{2}}=0$ is not invariant under Galilean transformation

## QUESTION FIVE [20 MARKS]

a. The position vectors of two particles are given respectively by $r_{1}=t i-t^{2} j+(2 t+3) k$ and $r_{2}=$ $\left(2 t-3 t^{2}\right) i+4 t j-t^{3} k$. Find the relative velocity and acceleration of the second with respect to the first at $\mathrm{t}=1$ second
b. Two trains on separate tracks move towards each other. Train 1 has speed of $130 \mathrm{~km} / \mathrm{hr}$ and train 2 has a speed of $90 \mathrm{~km} / \mathrm{hr}$. train 2 blows its horn, emitting a frequency of 500 Hz . What is the frequency heard by the engineer on train 1 [4 marks]
c. Use the relativistic Michelson-Morley experiment to prove that $\Delta t=\frac{D V^{2}}{C^{2}}$ where D is distance of separation

