TECHNICAL UNIVERSITY OF MOMBASA

FACULTY OF APPLIED AND HEALTH SCIENCES

MATHS AND PHYSICS DEPARTMENT

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

AMA 4435: MEASURE INTEGRATION AND PROBABILITY PAPER 2

END OF SEMESTER EXAMINATION

SERIES: MAY 2016

TIME: 2 HOURS

DATE: MAY 2016

Instructions to Candidates
You should have the following for this examination
-*Answer Booklet, examination pass and student ID*
This paper consists of FIVE questions. Attempt QUESTION 1 AND ANY OTHER TWO FROM QUESTIONS 2- 5.
Do not write on the question paper.

Question ONE (30 MARKS)

a. State three properties of a measure (3 marks)

b. Distinguish between the positive and negative parts of a function (4marks)

c. Let $(X, \mathcal{X})$ be a measurable space , if $\mathcal{X} \subseteq X$. when is f:$X \rightarrow R_e$ said to be measurable (3 marks)

d. Let $(X, \mathcal{X})$ be a measurable space. In order that a function f:$X \rightarrow R_e$ be $\mathcal{X}$- measurable. Outline the necessary and sufficient conditions that must be fulfilled (8marks)

e. Define the following terms as used in measure theory

   I. Simple function (2 marks)
   II. Characteristic function (2 marks)
   III. Probability measure (2 marks)
   IV. Complete measure (2marks)

f. State Fatou's lemma (4 marks)

Question TWO (20 marks)

a. Outline the necessary conditions for a function f to be integrable or summable (4marks)
b. Let (Ӿ, ӿ) be a measurable and f, g: Ӿ $\rightarrow \mathbb{R}_e$ be ӿ- measurable functions and let $c \in \mathbb{R}$. Prove that the functions $cf, c + f,\ f^2, |\ f|,\ f + g,\ fg\ f^+\ and\ f^-\ are\ all$ ӿ- measurable (16 marks)

Question THREE

a. Define a characteristic function (2marks)
b. Let E be a non Lebesque measure subset of (0, 1). Illustrate using a diagram a counter example to prove that$f \epsilon$ ӿ. (12 marks)
c. Prove that if a function f is measurable then a measurable function is integrable iff |f | is integrable and $|\int f du| \ll \int |f du|$ (8 marks)

Question FOUR (20 marks)

a. Let (Ӿ, ӿ, μ) be a Lebesque measurable space on $\mathbb{R}$ and let$f_n = \chi_{(0,n)}$, show that f_n converges uniformly to f but$\int f du \neq lim_{n\to\infty} \int f|du|$. Why does this not contradict the Monotone convergence theorem? Does Fatous Lemma apply? (14 marks)
b. State the law of large numbers. (2 marks)
c. What do you understand by the term probability measure? State four points. (4 marks)

Question FIVE (20 marks)

a. State Demorgan's laws and prove that a set is enclosed under countable intersections (6 marks)
b. prove that μ is monotone if $E, F \epsilon$ ӿ $and\ F \subset E$ (4 Marks)
c. State and prove the central limit theorem (10marks)