# TECHNICAL UNIVERSITY OF MOMBASA 

INSTITUTE OF COMPUTING AND INFORMATICS
Select department

# UNIVERSITY EXAMINATION FOR: <br> BACHELOR OF SCIENCE IN INFORMATION TECHNNOLOGY BIT 2212: BUSINESS SYSTEMS MODELING END OF SEMESTER EXAMINATION <br> SERIES:DECEMBER2016 <br> TIME:2HOURS 

DATE:Pick DateSelect MonthPick Year

## Instructions to Candidates

You should have the following for this examination
-Answer Booklet, examination pass and student ID
This paper consists of FIVE questions. Attemptquestion ONE (Compulsory) and any other TWO questions.
Do not write on the question paper.

## Question ONE

a) Explain the following terms:
i) Model

- Models represent reality
ii) Simulation
- Emulation of reality using a model
b) Explain three methods used to get information on objective reality
- Experimentation
- Analysis
- Simulation
c) Explain the classification of simulation models
- Continuous simulation: the state of the model changes continuously with the times.
- Discrete simulation: the state transition occurs at intervals, i. e. at discrete times.
d) A plastic company manufactures among other things, two types of containers X and Y . A number of processes are involved in the manufacture of both types. These processes take place within three different departments and the time required in each department for each pack are given in the following table.

Processing time per pack (Hours)
X Y

| Department A | 3 | 1 |
| :--- | :--- | :--- |
| Department B | 1 | 1 |
| Department C | 1 | 2 |

Departments A, B and C have 120, 60 and 100 machine hours respectively, available for the products. The contribution per pack is Sh 50 and Sh 80 for type X and Y respectively.

Formulate a linear programming model for the above problem.

Let the number of $X$ plastics be $x$ and the $Y$ plastics be $y$.
The objective function is to maximize the contributions from x and y :
e.g. Maximize $50 \mathrm{x}+80 \mathrm{y}$

Subject to the constraints:

$$
\begin{align*}
3 x+y & \leq 120  \tag{2}\\
x+y & \leq 60  \tag{3}\\
x+2 y & \leq 100  \tag{4}\\
x & \geq 0  \tag{5}\\
y & \geq 0 \tag{6}
\end{align*}
$$

e) Distinguish between solution derived from simulation models and solutions derived from analytical models


#### Abstract

ANS: In a business context the process of experimenting with a model usually consists of inserting different input values and observing the resulting output values.

Simulation is used where analytical techniques are not available or would be overly complex. Typical business examples are: most queuing systems other than the very simple, inventory control problems, production


## Question TWO

Faida Bank is in the process of devising a loan policy that involves a maximum of $\$ 12$ million. The following table provides the pertinent data about available types of loans.

| Types of Loans | Interest rate | Bad-debt ratio |
| :--- | :--- | :--- |
| Personal | .140 | .10 |
| Car | .130 | .07 |
| Home | .120 | .03 |
| Farm | .125 | .05 |
| Commercial | .100 | .02 |

Bad debts are unrecoverable and produce no interest revenue.

Competition with other financial institutions requires that the bank allocate at least $40 \%$ of the funds to farm and commercial loans. To assist the housing industry in the region, home loans must equal at least $50 \%$ of the personal, car, and farm loans. The bank also has a stated policy of not allowing the overall ratio of bad debts on all loans to exceed $4 \%$.

You are required to formulate a mathematical model for Thrift Bank.

## Mathematical model

The situation seeks to determine the amount of loan in each category, thus leading to the following definitions of the variables:

$$
\mathrm{X}_{1}=\text { Personal loans (in millions of dollars) }
$$

$$
\begin{align*}
& \mathrm{X}^{2}=\text { car loan } \\
& \mathrm{X}_{3}=\text { Home loans } \\
& \mathrm{X}_{4}=\text { farm loans } \\
& \mathrm{X}_{5}=\text { Commercial loans } \tag{2M}
\end{align*}
$$

The object of Faida bank is to maximise its net return, the difference between interest revenue and lost bad debt. The interest revenue is accrued only on loans in good standing. Thus, because $10 \%$ of personal loans are lost to bad debt, the bank will receive interest on only $90 \%$ of the loan - that is, it will receive $14 \%$ interest on $.9 x_{1}$ of the original loan $x_{1}$. The same applies to the other four types of loans.

$$
\begin{align*}
\text { Total interest } & =.14\left(.9 \mathrm{X}_{1}\right)+.13\left(.93 \mathrm{X}_{2}\right)+.12\left(.97 \mathrm{X}_{3}\right)+.125\left(.95 \mathrm{X}_{4}\right)+.1\left(.98 \mathrm{X}_{5}\right) \\
& =.126 \mathrm{X}_{1}+.1209 \mathrm{X}_{2}+.1164 \mathrm{X}_{3}+.11875 \mathrm{X}_{4}+.098 \mathrm{X}_{5} \tag{3M}
\end{align*}
$$

Also $\quad \mathrm{Bad}$ debt $=.1 \mathrm{X}_{1}+.07 \mathrm{X}_{2}+.03 \mathrm{X}_{3}+.05 \mathrm{X}_{4}+.02 \mathrm{X}_{5}$

Objective function
Maximise $\mathrm{Z}=$ Total interest - bad debt

$$
\begin{align*}
& =\left(.126 \mathrm{X}_{1}+.1209 \mathrm{X}_{2}+.1164 \mathrm{X}_{3}+.11875 \mathrm{X}_{4}+.098 \mathrm{X}_{5}\right)-\left(.1 \mathrm{X}_{1}+.07 \mathrm{X}_{2}+.03 \mathrm{X}_{3}+\right. \\
& \left..05 \mathrm{X}_{4}+.02 \mathrm{X}_{5}\right) \\
& =.026 \mathrm{X}_{1}+.0509 \mathrm{X}_{2}+.0864 \mathrm{X}_{3}+.06875 \mathrm{X}_{4}+.078 \mathrm{X}_{5} \tag{3M}
\end{align*}
$$

Constrains
a. Total funds should not exceed $\$ 12$ (milli0n)

$$
\begin{equation*}
X_{1}+X_{2}+X_{3}+X_{4}+X_{5} \leq 12 \tag{2M}
\end{equation*}
$$

ii) Farm and commercial loans equal at least $40 \%$ 0f all loans:

$$
\begin{align*}
& \mathrm{X}_{4}+\mathrm{X}_{5} \geq .4\left(\mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{3}+\mathrm{X}_{4}+\mathrm{X}_{5}\right) \\
& \text { Or } .4 \mathrm{X}_{1}+.4 \mathrm{X}_{2}+.4 \mathrm{X}_{3}-.6 \mathrm{X}_{4}-.6 \mathrm{X}_{5} \leq 0 \tag{2M}
\end{align*}
$$

iii) Home loans should equal at least $50 \%$ of personal, car and farm loans:

$$
X_{3} \geq .5\left(X_{1}+X_{2}+X_{4}\right)
$$

Or $\quad .5 \mathrm{X}_{1}+.5 \mathrm{X}_{2}-.5 \mathrm{X}_{3}+.5 \mathrm{X}_{4} \leq 0$
iv) Bad debts should not exceed $4 \%$ of all loans:

$$
.1 \mathrm{X}_{1}+.07 \mathrm{X}_{2}+.03 \mathrm{X}_{3}+.05 \mathrm{X}_{4}+.02 \mathrm{X}_{5} \leq .04\left(\mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{3}+\mathrm{X}_{4}+\mathrm{X}_{5}\right)
$$

Or

$$
\begin{equation*}
.06 \mathrm{X}_{1}+.03 \mathrm{X}_{2}-.01 \mathrm{X}_{3}+.01 \mathrm{X}_{4}-.02 \mathrm{X}_{5} \leq 0 \tag{2M}
\end{equation*}
$$

v) Non-negativity

$$
\begin{equation*}
X_{1} \geq 0, \quad X_{2} \geq 0, \quad X_{3} \geq 0, \quad X_{4} \geq 0, \quad X_{5} \geq 0 \tag{2M}
\end{equation*}
$$

An important assumption in the formulation is that all loans are issued at approximately the same time. Allowing us to r ignore differences in the time value of the funds allocated to the different loans.

## Question THREE

Three electric power plants with capacities of 25,40 and 30 million kWh supply electricity to three cities. The maximum demands at the three cities are estimated at 30,35 , and 25 million kWh . The price per million kWh at the three cities is given in the table below.

| Plant | City |  |  |
| :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 |
| 1 | $\$ 600$ | $\$ 700$ | $\$ 400$ |
| 2 | $\$ 320$ | $\$ 300$ | $\$ 350$ |
| 3 | $\$ 500$ | $\$ 480$ | $\$ 450$ |

During the month of August, there is a $20 \%$ increase in demand at each of the three cities, which can be met by purchasing electricity from another network at a premium rate of $\$ 1000$ per million kWh . The network is not linked to city 3 , however. The utility company wishes to determine the most economical plan for distribution and purchase of additional energy.
a) Formulate the problem as a transportation model
b) Determine an optimal distribution plan for the utility company (8 Marks)
c) Determine the cost of the additional power purchased by each of the three cities.

## Solution

( a and b ) using $\mathrm{M}=10,000$. Solution is in bold in the table below. Total cost $=\$ 49,710$

|  | 1 | 2 | 3 | Supply |
| :---: | :---: | :---: | :---: | :---: |
| Plant 1 | 600 | 700 | $\begin{aligned} & 400 \\ & 25 \end{aligned}$ | 25 |
| Plant 2 | $\begin{aligned} & 320 \\ & 23 \end{aligned}$ | $\begin{aligned} & 300 \\ & 17 \end{aligned}$ | 350 | 40 |
| Plant 3 | 500 | $\begin{aligned} & 480 \\ & 25 \end{aligned}$ | $\begin{aligned} & 450 \\ & 5 \end{aligned}$ | 30 |
| Excess <br> Plant 4 | $\begin{aligned} & 1000 \\ & 13 \end{aligned}$ | 1000 | M | 13 |
| Demand | 36 | 42 | 30 |  |

c) City 1 excess cost $=\$ 13,000$

## Question FOUR

Wild West produces two types of cowboy hats. A Type 1 hat requires twice as much labour time as a Type 2. If all the available time is dedicated to Type 2 alone, the company can produce a total of 400 Type 2 hats a day. The respective market limits for the two types are 150 and 200 hats per day. The revenue is $\$ 8$ per Type 1 hat and $\$ 5$ per Type 2 hat.
a) Use the graphical solution to determine the number of hats of each type that maximises revenue.
b) Determine the dual price of the production capacity (in terms of type 2 hat) and the range for which it is applicable.
c) If the daily demand for Type 1 hat is decreased to 120 , use the dual price to determine the corresponding effect on the optimal revenue.
d) What is the dual price of the market share of Type 2 hat? By how much can the market share be increased while yielding the computed worth per unit?

Let
$\mathrm{x}_{1}=$ number of Type 1 hats per day
$\mathrm{x}_{2}=$ number of Type 2 hats per day

Maximize $\mathrm{z}=8 \mathrm{x}_{1}+5 \mathrm{x}_{2}$ subject to:

$$
\begin{aligned}
& 2 x_{1}+x_{2} \leq 400 \\
& x_{1} \leq 150, x_{2} \leq 200 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

a)


$$
\mathrm{X}_{1}=100, \mathrm{x}_{2}=200, \mathrm{z}=\$ 1800 \text { at point } \mathrm{B} .
$$

b) $\$ 4$ per Type 2 hat in range $(200,500)$
c) No change because the dual price is $\$ 0$ per unit in the range $(100, \infty)$
d) $\$ 1$ worth per unit in the range $(100,400)$. Maximum increase $=200$ Type 2 .

## Question FIVE

a) Simulation techniques have been used to analyze problems of two distinct types: Practical Real life problems and theoretical problems related to basic sciences. Illustrate the statement giving examples of each type.

## Practical Real Life Examples

$\checkmark$ Simulation of industrial problems(maintenance schedule, queing problems)
$\checkmark$ Simulation in business and economic problems
$\checkmark$ Simulation of war strategies
Theoretical problems related to basic sciences like physics, chemistry and mathematics
b) A plant has a large number of similar machines. The machine breakdowns or failures are random and independent. The shift in-charge of the plant collected the data about the various machines breakdown times and the repair time required on hourly basis, and the record for the past 100 observations as shown below was:

| Time Between <br> Recorded <br> Machine <br> Breakdown <br> (hours) | Probability | Repair <br> Required <br> (hours) |  |
| :--- | :--- | :--- | :--- |
| 0.5 | 0.05 | 1 | Probability |
| 1 | 0.06 | 2 | 0.28 |
| 1.5 | 0.16 | 3 | 0.20 |
| 2 | 0.33 |  |  |
| 2.5 | 0.21 |  |  |
| 3 | 0.19 |  |  |

For each hour that one machine is down due to being or waiting to be repaired, the plant loses Rs. 70 by way of lost production. A repairman is paid at Rs. 20 per hour.
i) Simulate this maintenance system for 15 breakdowns.
ii) Obtain the total maintenance cost.

Use following pairs of random numbers:
$(61,87),(85,39),(16,28),(46,97),(88,69),(08,87),(82,52),(56,52),(22,15),(49,85) \quad(15$ Marks $)$
i) Machine breakdown look up table

| 0.5 | 0.05 | 0.05 | $5 \%$ | $0-4$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.06 | 0.11 | $11 \%$ | $5-10$ |
| 1.5 | 0.16 | 0.27 | $27 \%$ | $11-26$ |
| 2 | 0.33 | 0.60 | $60 \%$ | $27-59$ |
| 2.5 | 0.21 | 0.81 | $81 \%$ | $60-80$ |
| 3 | 0.19 | 1.0 | $100 \%$ | $81-99$ |

Repair look up table

| 1 | 0.28 | 0.28 | $28 \%$ | $0-27$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 0.52 | 0.80 | $80 \%$ | $28-79$ |
| 3 | 0.20 | 1.0 | $100 \%$ | $80-99$ |

Simulation table


ii) total maintenance cost - 2005

