**TECHNICAL UNIVERSITY OF MOMBASA**

**UNIVERSITY EXAMINATIONS 2016/2017**

**EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND COMPUTING**

**AMA 4429: OPERATIONS RESEARCH II**

**END OF SEMESTER EXAMINATIONS**

**SERIES: SEPT. 2017**

**DATE: SEPT. 2017** **DURATION:** 2 HOURS

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO**

**QUESTION ONE (30 MARKS)**

(a.) Give a brief explanation of the following terms:

(i.) Demand Matrix **(3 marks)**

(ii.) Production Matrix **(3 marks)**

(iii.) Open Input-Output Models **(3 marks)**

(iv.) Closed Input-Output Models **(3 marks)**

(b.) An assembly line contains 2,000 of a component which has a limited life. Records show that the life of the components is normally distributed with mean of 900 hours and standard deviation of 80 hours.

(i.) What proportion of components will fail before 1,000 hours? **(3 marks)**

(ii.) What proportion will fail before 750 hours? **(3 marks)**

(c.) Describe the recursive nature of Dynamic Programming **(4 marks)** (d.) From past experience it is known that a machine is set up correctly on 90% of occasions. If the machine is set up correctly then the conditional probability of a good part is 95% but if the machine is not set up correctly then the conditional probability of a good part is only 30%.

On a particular day the machine is set up and the first component produced and found to be good.

(i.) What is the probability that the machine is set up correctly? **(4 marks)**

(ii.) What is the probability that the machine is set up incorrectly? **(4 marks)**

**QUESTION TWO (20 MARKS)**

(a.) The states of a Markov Chain can be classified based on the transition probability $p_{ij}$ of **P.** Explain the following statements:

(i.) A state j is absorbing **(2 marks)**

(ii.) A state j is transient **(2 marks)**

(iii.) A state j is recurrent **(2 marks)**

(iv.) A state j is periodic **(2 marks)**

(b.) (i.) Define the steady-state probability in an Ergodic Markov Chain **(2 marks)**

(ii.) Explain the term “mean first return time” **(2 marks)**

(c.)(i.) Determine the steady-state probability distribution, given the transition matrix

$$P = \begin{pmatrix} 0.3 & 0.6 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0.05 & 0.4 & 0.55 \end{pmatrix}$$

**(4 marks)**

(ii.) Compute the mean first return time **(4 marks)**

**QUESTION THREE (20 MARKS)**

(a.) Give the three basic elements of the Dynamic Programming Model **(3 marks)**

(b.) Describe the Knapsack/Fly-Away kit/Cargo-Loading Model **(6 marks)**

(c.) A 4-ton vessel can be loaded with one or more of three items. The following table gives the unit weight, $w_i$, in tons and the unit revenue in thousands of dollars, $r_i$, for item *i*.

| Item $i$ | $w_i$ | $r_i$ |
|---|---|---|
| 1 | 2 | 31 |
| 2 | 3 | 47 |
| 3 | 1 | 14 |

Determine the number of units of each item that will maximize the total return **(11 marks)**

**QUESTION FOUR (20 MARKS)**

(a.) Describe the shortest-route problem **(2 marks)**

(b.) Describe the Dijkstra's Algorithm for determining the shortest routes between the source node and every other node in the network **(8 marks)**

(c.) The network in Figure 1-1gives the permissible routes and their lengths in kilometres between city 1 and four other cities (nodes 2 to 5).

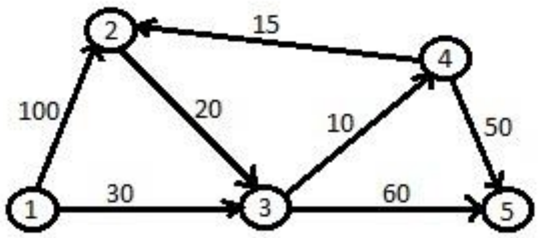

Fig 1-1: Network for Dijkstra's shortest-route algorithm

Determine the shortest routes between city 1 and each of the remaining four cities

**(10 marks)**

**QUESTION FIVE (20 MARKS)**

(a.) Describe the general constrained nonlinear programming problem **(4 marks)**

(b.) Describe the separable programming as a method to solve non-linear problems **(4 marks)**

(c.) Use separable programming to solve the problem

Maximize $z = x_1 + x_2^4$

subject to

$$3x_1 + 2x_2^2 \leq 9$$
$$x_1, x_2 \geq 0$$

**(12 marks)**